Tečné a normálové zrychlení - odvození

$$\vec{\tau}^0 \equiv \frac{d\vec{r}}{|d\vec{r}|} = \frac{d\vec{r}}{ds} = \vec{r}\,' \qquad \vec{r} = \vec{r}(s) = \vec{r}(s(t))$$

$$\vec{v} = \frac{d\vec{r}(s(t))}{dt} = \frac{d\vec{r}}{ds}\frac{ds}{dt} = \vec{r}\,'\frac{ds}{dt} = \vec{\tau}^0\frac{ds}{dt} \qquad \vec{\tau}^0 = \vec{\tau}^0(s(t))$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\vec{\tau}^0\right) = \frac{d^2 s}{dt^2}\vec{\tau}^0 + \frac{ds}{dt}\frac{d\vec{\tau}^0}{dt} = \frac{d}{dt}\frac{ds}{dt}\vec{\tau}^0 + \frac{ds}{dt}\frac{d\vec{\tau}^0}{ds}\frac{ds}{dt} =$$

$$v = \sqrt{\vec{v}.\vec{v}} = \sqrt{\left(\frac{ds}{dt}\right)^2 \vec{\tau}^0.\vec{\tau}^0} = \frac{ds}{dt} \qquad x\,' = \frac{dx}{ds}$$

$$= \frac{dv}{dt}\vec{\tau}^0 + v^2\vec{\tau}\,' \qquad = \frac{dv}{dt}\vec{\tau}^0 + v^2\frac{\vec{\nu}^0}{R}$$

$$(\vec{\tau}^0)^2 = \vec{\tau}^0.\vec{\tau}^0 = \tau_1^2 + \tau_2^2 + \tau_3^2 = 1$$

$$2\tau_1\tau_1' + 2\tau_2\tau_2' + 2\tau_3\tau_3' = 0$$

$$\vec{\nu}^0 = \frac{\vec{\tau}\,'}{|\vec{\tau}\,'|} = \frac{\vec{\tau}\,'}{\sqrt{\vec{\tau}\,'.\vec{\tau}\,'}} = R\vec{\tau}\,' \qquad \frac{1}{\sqrt{\vec{\tau}\,'.\vec{\tau}\,'}} = R$$

$$\vec{\tau}\,'.\vec{\tau}^0 = 0$$

1. Frenetův vzorec