Uncertainties of measurement

There are two methods for determination of uncertainty.

Method A – based on mathematical statistics
Method B – based on information given by measuring devices

Method A

This method uses mathematical statistics. An average value is calculated first by the formula.
N is number of measurements
x_i – i-th sample

$$\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i,$$

Then the standard uncertainty equal to the standard deviation of average value is given by

$$u_A = \overline{s} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N(N-1)}}.$$

Method B

For successful calculation of this method we need some data from measuring devices, mostly their precision or resolution. If the resolution of the device is **Δ** (which means one basic division at a needle device, for example), then the B type uncertainty is given by

$$u_B = \frac{\Delta}{\sqrt{12}}.$$

<u>Caliper</u>
Δ = 20 um,

$$u_B = \frac{20\,\mu\mathrm{m}}{\sqrt{12}} \approx 5{,}8\,\mu\mathrm{m}.$$

<u>Micrometer</u>
Δ = 10 um

u_B= 2,9 um

<u>Needle measuring device</u>
This device is characterized by a *class of precision* CP and its *range* R.
Class B uncertainty is given by u_B= (2x(R*CP)/100) / sqrt(12) or uB= ((R*CP)/100) / sqrt(3)

Example: CP= 0,5 % ; R= 600 mA

$$\pm 600 \times \frac{0{,}5}{100}\,\mathrm{mA} = \pm 3\,\mathrm{mA}.$$

$$u_B = \frac{3}{\sqrt{3}}\,\mathrm{mA} \approx 1{,}7\,\mathrm{mA}.$$

Digital measuring device

Precision is typically given by expression **p**% of rdg + **n** digits

u_B= ((p*rdg)/100 + ls digit value) / sqrt(3)

Example: digital multimeter, full range is 19,99 V; p= 0,5% ; rdg= 12,69V; n= 1 digit; ls digit value= 0,01 V

$$\pm\left(12{,}69 \times \frac{0{,}5}{100} + 0{,}01\right)\ \mathrm{V} = \pm 0{,}163\ \mathrm{V}.$$

$$u_{\mathrm{B}} = \frac{0{,}163}{\sqrt{3}}\ \mathrm{V} \approx 94\ \mathrm{mV}.$$

Digital device with unknown precision

Precision is commonly considered 2 times the least significant digit

Example: digital counter measuring time, display format is xxxx.xx seconds. The least significant digit is 0,01 s => Δ= 0,02 s

u_B = Δ / sqrt(12) = 0,02 / sqrt (12) = 0,0058 s = 5,8 ms

Time measurement by a stopwatch

A student will estimate his/her precision os measurement.

Example Δ= 0,3 s

u_B = Δ / sqrt(12) = 0,3 / sqrt (12) = 0,087 s = 87 ms

Combined uncertainty for direct measurement – C type uncertainty

Is given by a formula

$$u_{\mathrm{C}} = \sqrt{u_{\mathrm{A}}^2 + u_{\mathrm{B}}^2}.$$

Combined uncertainty for a function

$$Z = f(X_1, X_2, \ldots, X_M)$$

$$u_c^2(Y) = \sum_{i=1}^{M} \left(\frac{\partial f}{\partial X_i}\right)^2_{\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_M} u^2(X_i).$$

$$h = \frac{1}{2} g t^2 \qquad \Rightarrow \qquad g = \frac{2h}{t^2},$$

$$\overline{g} = \frac{2\overline{h}}{\overline{t}^2}.$$

$$\frac{\partial g}{\partial h} = \frac{2}{t^2}, \qquad \frac{\partial g}{\partial t} = -\frac{4h}{t^3}$$

$$u(g) = \sqrt{\frac{4}{\overline{t}^4} u^2(h) + \frac{16\overline{h}^2}{\overline{t}^6} u^2(t)}.$$