3. GRAVITATION

3 - 1 Newton's Law of Universal Gravitation

The law of universal gravitation states as follows:

> Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

The magnitude of this force can be written as

$$F = G \frac{m_1 m_2}{r^2}, \qquad (3\text{-}1)$$

where m_1 and m_2 are the masses of the two particles, r is the distance between them and G is a universal constant that is measured experimentally and has the same numerical value for all objects:

$$G \doteq 6.67 . 10^{-11} \ N.m^2/kg^2.$$

Eq. 3-1 gives us the magnitude of the gravitational force. When bodies are small compared to the distance between them, we may assume them to be particles and we obtain only little inaccuracy results.

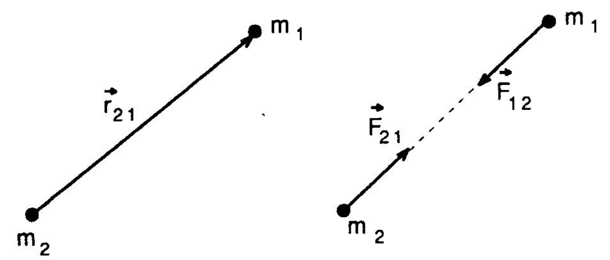

Figure 3 - 1

We can also write the law of universal gravitation in vector form as (observe Fig. 3-1):

$$\vec{F}_{12} = - G \frac{m_1 m_2}{r_{21}^2} \vec{r}^{\,o}_{21}. \qquad (3\text{-}2)$$

Where $\vec{F}_{12}$ is the vector force on particle 1 of mass m_1 exerted by particle 2 of mass m_2. As $\vec{r}_{21}$ is the displacement vector that points from particle 2 toward particle 1, so $\vec{r}^{\,o}_{21} = \vec{r}_{21}/r_{21}$ is a unit vector that points from particle 1 toward particle 2 along the line joining them. The minus sign in Eq. 3-2 is necessary because the force $\vec{F}_{12}$ on particle 1 points in the direction opposite to $\vec{r}^{\,o}_{21}$.

Since $\vec{r}_{12} = - \vec{r}_{21}$, by the third law of motion, the force $\vec{F}_{21}$ acting on m_2 exerted by m_1 must have the same magnitude as $\vec{F}_{12}$ but acts in the opposite direction:

$$\vec{F}_{21} = - \vec{F}_{12} = G \frac{m_1 m_2}{r_{21}^2} \vec{r}^{\,o}_{21} = - G \frac{m_1 m_2}{r_{12}^2} \vec{r}^{\,o}_{12}.$$

3 - 2 Gravity Near the Earth's Surface

Let us apply Eq. 3-1 to the gravitational force between the earth and an object at its surface. We put m_1 equals to the mass M of the earth and r the distance from the earth's center which equals the radius R of the earth.

This force of gravity due to the earth is the weight mg of the object; thus

$$mg = G\frac{mM}{R^2}$$

and the acceleration of gravity at the surface of the earth is

$$g = G\frac{M}{R^2}. \qquad (3\text{-}3)$$

This equation does not give precise value for g at various locations because the earth is not a perfect sphere; its rotation also has an effect on the measurement of g.

3 - 3 Satellites

For satellites that move (approximately) with uniform circular motion, the acceleration is v^2/r. The force that gives a satellite this acceleration is the force of gravity given by Eq. 3-1.

When we apply the second law of motion F = ma, we have

$$G\frac{mM}{r^2} = m\frac{v^2}{r}, \qquad (3\text{-}4)$$

where r is the distance of the satellite from the earth's center and v is its speed.

From Eq. 3-4 we can calculate the orbital velocity required for a satellite to stay in a orbit h km above the earth's surface:

$$v = \sqrt{\frac{GM}{r}},$$

where M is the mass of earth and r = R + h (R is the radius of the earth). Note that the orbital velocity decreases with increasing height h .

We can also determine period T of the satellite (time to make one orbit):

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}.$$

3 - 4 Kepler's Laws

Kepler's laws describe the planetary motion. These are summarized as follows:

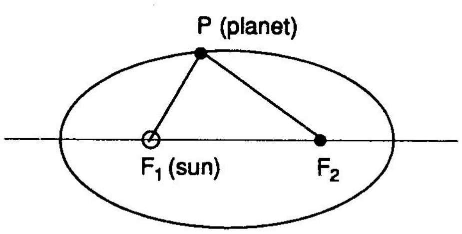

Figure 3 - 2

Kepler's first law: the path of each planet about the sun is an ellipse with the sun at one focus.

We know that an ellipse is a closed curve such that the sum of the distances from any point P on the curve to two fixed points, called the foci F_1 and F_2, remains constant. That is, the sum of the distances $F_1P + F_2P$ is the same for all points P on the curve.

Kepler´s second law: each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal periods of time.

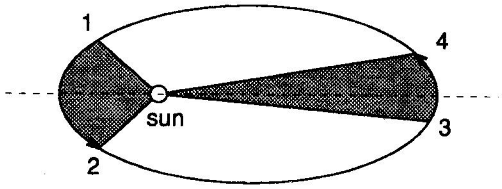

Figure 3 - 3

Planets move fastest in that part of orbit where they are closest to the sun.

Kepler´s third law: the ratio of the squares of the periods of any two planets is equal to the ratio of the cubes of their semimajor axis. That is

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3,$$

where T_1 and T_2 represent the periods of two planets, r_1 and r_2 their semimajor axis.

Example: Derive Kepler´s third law from the special case of a circular motion.

Solution: We start from the second law of motion; for F we substitute the law of universal gravitation and for a the centripetal acceleration, so

$$F = ma$$

or

$$G\frac{m_1 M}{r_1^2} = m_1 \frac{v_1^2}{r_1}. \qquad (3\text{-}5)$$

Here m_1 is the mass of a planet, r_1 is its orbit radius, v_1 is the speed of the planet in orbit, M is the mass of the sun.
The period of its motion is

$$T_1 = \frac{2\pi r_1}{v_1}. \qquad (3\text{-}6)$$

We substitute for v_1 from Eq. 3-6 into Eq. 3-5:

$$G\frac{m_1 M}{r_1^2} = m_1 \frac{4\pi^2 r_1}{T_1^2}$$

and

$$\frac{T_1^2}{r_1^3} = \frac{4\pi^2}{GM}.$$

The same result we may derive for a second planet:

$$\frac{T_2^2}{r_2^3} = \frac{4\pi^2}{GM},$$

where T_2 and r_2 are the period and orbit radius respectively, for the second planet.

From last two results we obtain

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3,$$

which is Kepler's third law.

4. WORK AND ENERGY

The present chapter is devoted to the very important concept of energy and the closely related concept of work. These two quantities are scalars and so have no direction. Energy derives its importance from two sources. First, it is a conserved quantity and second, energy is a concept that is useful not only in the study of motion but in all areas of physics and in other sciences as well.

4 - 1 Work done by a Constant Force

Definition: the work done on a particle by a constant force (constant in both magnitude and direction) is defined as the product of the magnitude of the displacement times the component of the force parallel to the displacement.

So,

$$W = F x \cos\varphi, \qquad (4\text{-}1)$$

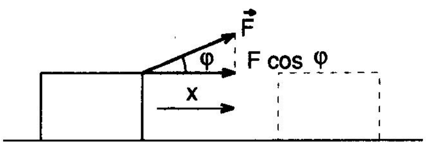

Figure 4 - 1

where F is the constant force, x is the net displacement of the particle and φ is the angle between the directions of the force and the net displacement. Let us note the factor $F\cos\varphi$ is the component of the force parallel to the displacement (see Fig. 4-1).

Simple case occurs when the motion and the force are in the same direction, so that $\cos\varphi = 1$ and $W = Fd$.

The unit of work is the joule (J): J = N m.

Example : A box of mass m is pulled on distance x along a horizontal floor by a constant force F_p which acts at angle φ. The floor is rough and exerts a friction force F_f.

Determine the work done by each force acting on the box, and the net work done on the box.

Solution: There are four forces acting on the box as shown in Fig. 4-2; the pull force F_p, the friction force F_f, the weight of box mg and the normal force N exerted upward by the floor.