$$\sigma = \frac{F}{A} . \tag{7-9}$$

We define the strain ε as the ratio of the change in length to the original length

$$\varepsilon = \frac{\Delta L}{L_o} . \tag{7-10}$$

It is the fractional change in length of an object. Now Eq. (7-8) can be rewritten as

$$\frac{F}{A} = E \frac{\Delta L}{L_o} ,$$

or

$$\sigma = E \varepsilon . \tag{7-11}$$

We see that the strain is directly proportional to the stress. This dependence is called Hook's law.

Compressive stress is the exact opposite of tensile stress; the material is compressed. Eq. (7-11) apply equally well to compression and tension, and the values for E are also the same.

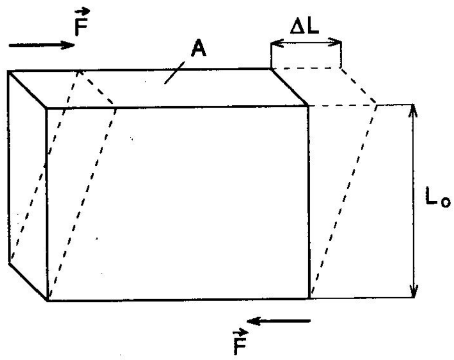


Figure 7-6

The third type of the deformation is shear stress. An object under shear stress has equal and opposite forces applied across its opposite faces. The shape of the object does change as shown in Fig. 7-6.

An equation similar to (7-8) can be applied to calculate shear strain:

$$\Delta L = \frac{1}{G} \frac{F}{A} L_o , \tag{7-12}$$

where A is the area of the surface parallel to the applied force (not perpendicular as for tension) and ΔL is perpendicular to L_o. The constant of proportionality G is called the shear modulus and is generally one-third to one-half the value of the elastic modulus E.

As the forth type of deformation we assume a body submerged in a fluid. In this case the fluid exerts a pressure on the object in all directions, as we shall see in chapter 8. Pressure is defined as force per unit area and thus is the equivalent of stress. In this case the fractional change in volume dV/V_o of an object is proportional to the increase in the pressure dp:

$$\frac{dV}{V_o} = - \frac{1}{K} dp , \tag{7-13}$$

where dV is the change of the volume, V_o is the original volume, dp is the increase in the pressure and K is proportionality constant called the bulk modulus. Since liquids and gases do not have a fixed shape, only the bulk modulus applies to them, and not the shear or Young's modulus. The minus sign in Eq. (7-13) indicates that the volume decreases with an increase in pressure.

8. FLUIDS AT REST

It is known that a liquid cannot keep a fixed shape. It takes on the shape of its container, but like a solid it is not readily compressible and its volume can be changed significantly only by a very large force. A gas has neither a fixed shape nor a fixed volume, it will expand to fill its container. Since liquids and gases do not keep a fixed shape, they each have the ability to flow; they are thus often called collectively as fluids. In this chapter we will discuss the behavior of fluids at rest.

8 - 1 Pressure in Fluids

First, we define the density ρ of a substance as its mass per unit volume

$$\rho = \frac{m}{V},$$

where m is the mass of the substance whose volume is V.

The SI unit for density is kg/m^3.

Pressure P is defined as force per unit area, where the force F is understood to be acting perpendicular to the surface area A :

$$P = \frac{F}{A}. \qquad (8\text{-}1)$$

The SI unit of pressure is N/m^2 which has the name pascal (Pa): Pa = N/m^2.

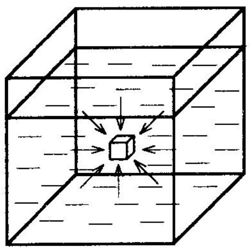


Figure 8 - 1

The concept of pressure is particulary useful in dealing with fluids. It is an experimental fact that a fluid exerts a pressure in all directions. At a particular point in a fluid at rest the pressure is the same in all directions - see Fig. 8-1; if it weren't the fluid would be in motion.

Another important property of a fluid at rest is that the force due to fluid pressure always acts perpendicularly to any surface which is in contact with.

Let us now calculate quantitatively how the pressure in a liquid of uniform density varies with depth. Consider a point at a depth h below the surface of the liquid, as shown in Fig. 8-2. The pressure due to the liquid at this depth h is due to the weight of the column of liquid above it. Thus the force acting on the area A is $F = mg = \rho Ahg$, where Ah is the volume of the column, ρ is the density of the liquid, and g is the acceleration of gravity. The pressure P is then

$$P = \frac{F}{A} = \frac{\rho Ahg}{A} \qquad (8\text{-}2)$$

and

$$P = \rho gh.$$

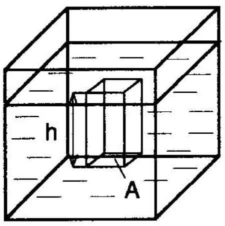


Figure 8 -2

Thus the pressure is directly proportional to the density of the liquid and to the depth within the liquid. In general, the pressure at equal depths is the same. Eq. 8-2 tells us what the pressure is at depth h in the liquid due to the liquid itself.

Next we consider the general case of determining how the pressure in a fluid varies with depth. We will want to determine the pressure at any height y above some reference point, as shown in Fig. 8-3.

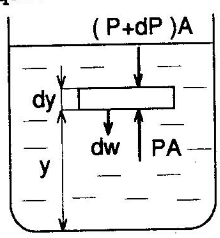


Figure 8 - 3

Within this fluid at the height y we consider a tiny, flat volume of the fluid whose area is A and whose thickness is dy .

Let the pressure acting upward on its lower surface at height y be P. The pressure acting downward on the top surface is $P + dP$. So the fluid pressure exerts a force PA upward and a force $(P + dP)A$ downward. The other force acting vertically is the force of gravity dW

$$dW = dm\, g = \rho\, dV\, g = \rho g A\, dy ,$$

where ρ is the density of the fluid at the height y . Since the fluid is assumed to be at rest, our considered volume is in equilibrium, so the net force on it must be zero:

$$PA - (P + dP)A - \rho g A\, dy = 0$$

or

$$\frac{dP}{dy} = -\rho g . \qquad (8\text{-}3)$$

This relation tells us how the pressure varies with height within the fluid. The minus sign indicates that the pressure decreases with an increase in height.

If the pressure at height y_1 is P_1, at height y_2 it is P_2, we can integrate Eq. (8-3) to obtain

$$\int_{P_1}^{P_2} dP = -\int_{y_1}^{y_2} \rho g\, dy ,$$

$$P_2 - P_1 = -\int_{y_1}^{y_2} \rho g\, dy . \qquad (8\text{-}4)$$

For liquids in which ρ = constant, Eq. (8-4) can be readily integrated:

$$P_2 - P_1 = -\rho g (y_2 - y_1) . \qquad (8\text{-}5)$$

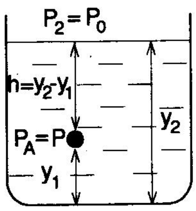


Figure 8 - 4

For the everyday situation of a liquid in an open container (such as water in a glass, a swimming pool, a lake, the ocean) there is a free surface at the top. And it is convenient to measure distances from this top surface; that is, we let h be the depth in the liquid where $h = y_2 - y_1$ as shown in Fig. 8-4.

If we let y_2 be the position of the top surface, then P_2 represents atmospheric pressure P_o at the top surface. So, from Eq. (8-5) the pressure P $(= P_1)$ at depth h in the fluid is

$$P = P_o + \rho g h . \qquad (8\text{-}6)$$

Note that Eq. (8-6) is Eq. (8-2) for liquid pressure plus the pressure P_o due to the atmosphere above.

8 - 2 Pascal's Principle

It is known that earth's atmosphere exerts a pressure on all objects with which it is in contact, including fluids. Atmospheric pressure acting on a fluid is transmitted throughout that fluid. We have known the water pressure at a depth h below the surface of a lake is $P = \rho g h$. The total pressure at this point is due to the pressure of water plus the pressure of the air above it. This is just one example of a general principle called Pascal's principle which states that pressure applied to a confined fluid increases the pressure throughout the fluid by the same amount.

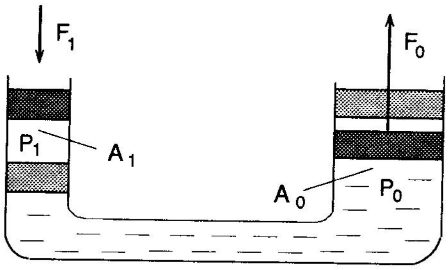


Figure 8 - 5

As a practical device making use of this principle we see in Fig. 8-5. Here it is illustrated the hydraulic lift - a small force can be used to exert a large force by making the area of one piston (the output) larger than the area of the other (the input).

According to Pascal's principle

$$P_o = P_1$$

or

$$\frac{F_o}{A_o} = \frac{F_1}{A_1}$$

and so

$$\frac{F_o}{F_1} = \frac{A_o}{A_1} . \qquad (8\text{-}7)$$

8 - 3 B u o y a n c y   a n d   A r c h i m e d e s' P r i n c i p l e

It is known that object submerged in a fluid appear to weigh less than it does when outside the fluid. Many objects such as wood float on the surface of the water. These are two examples of buoyancy. In both examples, the force of gravity is acting downward, but in addition, an upward buoyant force is exerted by the liquid.

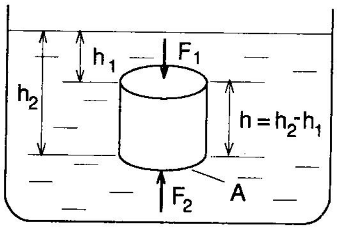


Figure 8 - 6

The buoyant force arises from the fact that the pressure in fluid increases with depth. Thus the upward pressure on the bottom surface of a submerged object is greater than the downward pressure on its top surface. For example, we may consider a cylinder of height h whose top and bottom have an area A and which is completely submerged in a fluid of density ρ_f, as shown in Fig. 8-6. The fluid exerts a pressure $P_1 = \rho_f g h_1$ on the top surface of the cylinder; the force due to this pressure on top of the cylinder is $F_1 = P_1 A = \rho_f g h_1 A$ and it is directed downward. Similarly, the fluid exerts an upward force on the botton of the cylinder equal to $F_2 = P_2 A = \rho_f g h_2 A$.

Since $F_2 > F_1$, the net force due to the fluid pressure, which is called the buoyant force, acts upward and has the magnitude

$$F = F_2 - F_1 = \rho_f g A(h_2 - h_1) = \rho_f g A h = \rho_f g V , \qquad (8\text{-}8)$$

where $V = Ah$ is the volume of the cylinder. Since ρ_f is the density of the fluid, the product $\rho_f g V = m_f g$ is the weight of fluid which takes up a volume equal to the volume of the cylinder. Thus the buoyant force on the cylinder is equal to the weight of the fluid displaced by the cylinder. This result is of course valid no matter what the shape of the submerged object and it is called Archimedes' principle.

E x a m p l e 1 : A 70-kg rock lies at the bottom of a lake. Its volume is 3.10^{-2} m^3. How much force is needed to lift it?

Solution: The buoyant force on the rock is equal

$$F = \rho_{H_2O}\, gV = 10^3 \text{ kg m}^{-3}.9{,}8 \text{ m.s}^{-2}.3.10^{-2} \text{ m}^3 = 2{,}9.10^2 \text{ N}.$$

The weight of the rock is $mg = 70 \text{ kg} \cdot 9{,}8 \text{ m s}^{-2} = 6{,}9 \cdot 10^2$ N. Hence the force needed to lift it is 690 N - 290 N = 400 N. It is as if the rock had a mass of only 400 N/9,8 m s^{-2} = 41 kg.

Archimedes' principle also applies well to objects that float, such as wood. In general we can say that an object floats on a fluid if its density is less than that of the fluid. For example, a log whose density is 600 kg m^{-3} and whose volume is 2 m^3 will have a mass of 1200 kg. If the log is fully submerged, it will displace a mass of water $m = \rho V = 1000 \text{ kg m}^{-3} \cdot 2 \text{ m}^3 = 2000$ kg. Hence the buoyant force on it will be greater than its weight, and it will float to the top. It will come the equilibrium when it displaces 1200 kg of water, which means that 1,2 m^3 or 0,6 of its volume will be submerged. In general we can say that the fraction of the object submerged is given by the ratio of the object's density to that of the fluid.

Air is a fluid and it also exerts a buoyant force. Ordinary objects weigh less in air than they do if weighed in a vacuum. Because the density of air is so small, the effect for ordinary solids is slight.

Example 2 : There are objects that float in air - for example helium balloons. Let us calculate what volume of helium is needed if a balloon is to lift a load of 800 kg (including the weight of the empty balloon). Assume $\rho_{air} = 1{,}29 \text{ kg m}^{-3}$, $\rho_{He} = 0{,}18 \text{ kg m}^{-3}$.

Solution: The buoyant force on the helium which is equal to the weight of displaced air, must at least be equal to the weight of the helium plus the load:

$$F = (m_{He} + 800) \cdot g ,$$

or

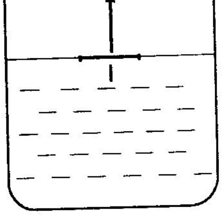
$$\rho_{air} Vg = (\rho_{He} V + 800) \cdot g .$$

Solving for V we have

$$V = \frac{800}{\rho_{air} - \rho_{He}} = \frac{800}{1{,}29 - 0{,}18} = 720 \text{ m}^3 .$$

8 - 4 Surface Tension

A number of common observations suggest that the surface of a liquid acts like a stretched membrane under tension. For example, a steel needle can be made to float on the surface of water even though it is denser than the water. The surface of a liquid acts like it is under tension, and this tension, acting parallel to the surface, arises from the attractive forces between the molecules. This effect is



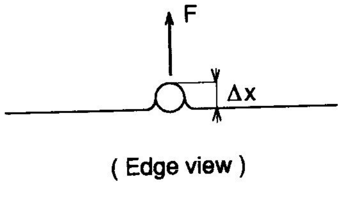


Figure 8 - 7

called surface tension γ , that is defined as the force F per unit length that acts a cross any line in a surface:

$$\gamma = \frac{F}{\ell} \cdot \qquad (8-9)$$

Let us consider a steel needle floating on the surface of water (Fig. 8-7).

To pull a needle up a force F is required and thus the surface tension γ is

$$\gamma = \frac{F}{2\ell}, \qquad (8-10)$$

so the unit of the surface tension is $[\gamma] = N\ m^{-1} = kg\ s^{-2}$. The temperature has a considerable effect on surface tension.

We can see how surface tension arises from the molecular point of view. The molecules of a liquid exert attractive forces on each other; these attractive forces are shown acting in Fig. 8-8 on a molecule deep within the liquid and on a second molecule at the surface. The molecule inside the liquid is in equilibrium due to the forces of other molecules acting in all directions. The molecule at the surface is also normally in equilibrium (the liquid is at rest); this is true even though the forces on a molecule at the surface can only be exerted by molecules below it (or at an equal height). Hence there is a net attractive force downward, which tends to compress the surface layer slightly - but only to the point where this downward force is balanced by an upward (repulsive) force due to close contact (or collision with) the molecules below. This compression of the surface means that, the liquid tries to minimize its surface area. This is why water tends to form spherical droplets, for a sphere represents the minimum possible surface area for a given volume.

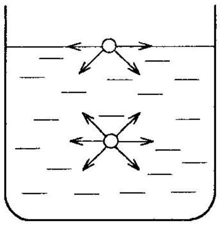


Figure 8 - 8

In order to increase the surface area of a liquid, a force is required and work must be done to bring molecules from the interior to the surface. This work increases the potential energy of the molecules and is sometimes called surface energy. The greater the surface area, the greater the surface energy.

The amount of work needed to increase the surface area can be calculated from Fig. 8-7 and Eq. (8-10)

$$\begin{aligned} W &= F\Delta x \\ &= \gamma 2\ell\Delta x \\ &= \gamma\Delta A, \end{aligned}$$

where Δx is the change in distance and ΔA is the total increase in area (at both surfaces in Fig. 8-7). So we can write

$$\gamma = \frac{W}{\Delta A}.$$

Thus, the surface tension γ is not only equal to the force per unit length; it is also equal to the work done per unit increase in surface area. Hence, γ can be specified in N/m or J/m^2.