9. FLUIDS IN MOTION

9-1 Flow Rate and the Equation of Continuity

We can distinguish two main types of fluid flow. If the flow is smooth, such that neighboring layers of the fluid slide by each other smoothly, the flow is said to be streamline or laminar flow. In this kind of flow, each particle of the fluid follows a smooth path, and these paths do not cross over one another (Fig. 9-1a). Above a certain speed the flow becomes turbulent. Turbulent flow is characterized by erratic, small whirlpool like circles called eddy currents or eddies. (Fig. 9-1b.) Eddies absorb a great deal of energy.

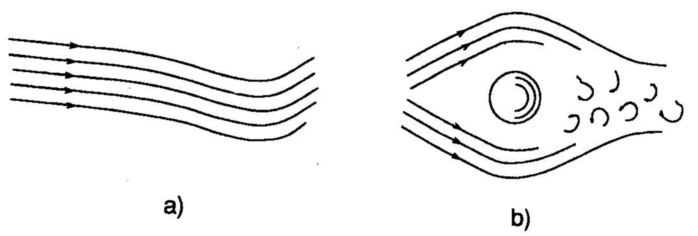

Figure 9 - 1

The flow can be steady, which means the velocity of the fluid at each point in space remains constant in time (which does not imply that the velocity is the same at all points in space).

In the steady, laminar flow of a fluid, the path taken by a given particle is called a streamline (see Fig. 9-1a). The fluid velocity at any point is tangent to the streamline at that point. Two streamlines cannot cross over one another, since this would imply that at such a cross point the velocity would not be unique. A bundle of streamlines, such as those shown in Fig. 9-2 is called a tube of flow. Since the streamlines represent the paths of particles, we see that no fluid can flow into or out of the sides of a tube of flow.

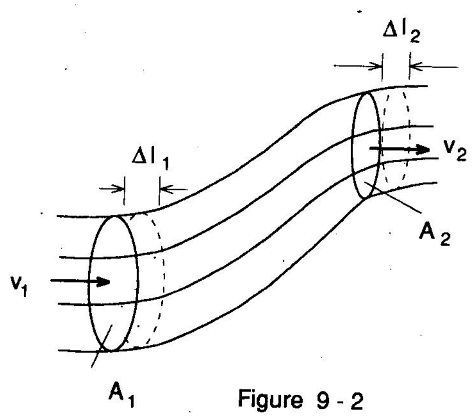

Figure 9 - 2

We now examine the steady streamline flow of a tube of flow and determine how the speed of the fluid varies with the size of the tube. Let us assume the tube small enough so that the velocity at any cross section is essentially constant (see Fig. 9-2). v_1 represents the fluid's velocity at the cross area A_1, v_2 at A_2. The mass flow rate is defined as the mass that passes any cross area per unit time. In Fig. 9-2 the volume of fluid passing through area A_1 in a time Δt is just $A_1 \Delta \ell_1$, where $\Delta \ell_1$ is the distance the fluid moves in time Δt. The velocity of fluid here is $v_1 = \Delta \ell_1 / \Delta t$. So, the mass flow rate through area A_1 is

$$\frac{\Delta m}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta \ell_1}{\Delta t} = \rho_1 A_1 v_1 ,$$

ρ_1 is the fluid density. Similarly we can write for area A_2

$$\frac{\Delta m}{\Delta t} = \rho_2 A_2 v_2 .$$

Since no fluid flows in or out the sides of a tube of flow, the flow rate through A_1 and A_2 must be equal. Thus

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 . \tag{9-1}$$

This is called the equation of continuity.

If the fluid is incompressible, which is a good approximation for liquids under most circumstances, then $\rho_1 = \rho_2$ and the equation (9-1) becomes

$$A_1 v_1 = A_2 v_2 . \tag{9-2}$$

This result tells us that where the cross area of a flow tube is large, the velocity is low and where the area is small, the velocity is high.

9 - 2 Bernoulli's Equation

We will assume the incompresible fluid whose flow is stedy and laminar. We consider a tube of flow which varies in cross section and also in height above some reference level (as shown in Fig. 9-3).

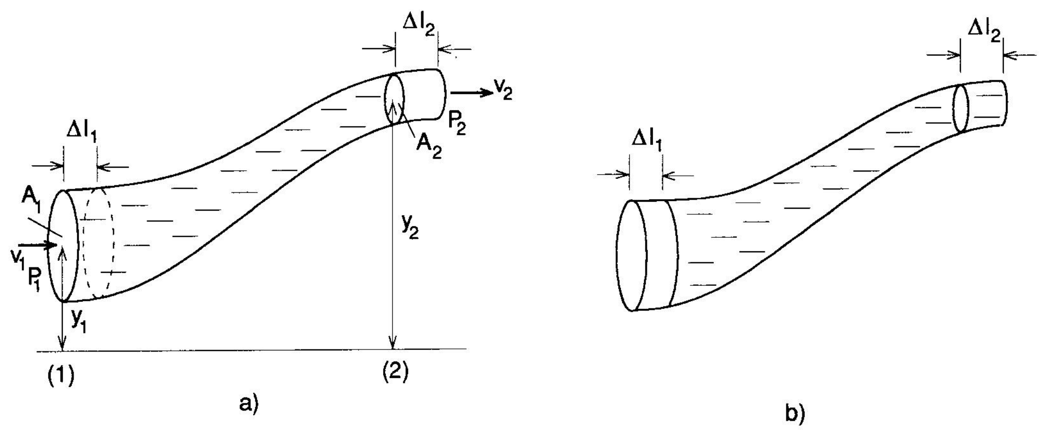

Figure 9 - 3

Let us consider the amount of fluid between area A_1 and A_2 and calculate the work done to move it from the position shown in (a) to that shown in (b). The fluid at point (1) moves a distance $\Delta \ell_1$, at the point (2) a distance $\Delta \ell_2$. The fluid to the left of point (1) exerts a pressure P_1 on the assumed fluid and does an amount of work $W_1 = F_1 \Delta \ell_1 = P_1 A_1 \Delta \ell_1$. At point (2) the work done is $W_2 = - P_2 A_2 \Delta \ell_2$; the negative sign is present because the force exerted on the fluid is opposite to the motion.

Work is also done on the fluid by the force of gravity. Since the net effect of all the process in Fig. 9-3 is to move a mass of volume $A_1 \Delta \ell_1$ $(= A_2 \Delta \ell_2)$ from point (1) to point (2), the work done by gravity is

$$W_3 = - mg(y_2 - y_1) ,$$

which is negative since the motion is against the force of gravity.

The net work W done on the fluid is thus

$$W = W_1 + W_2 + W_3 = P_1 A_1 \Delta \ell_1 - P_2 A_2 \Delta \ell_2 - mgy_2 + mgy_1 .$$

According to the work-energy theorem (see section 4-4), the net work done on the fluid must be equal to its change in kinetic energy; so

$$P_1A_1\Delta \ell_1 - P_2A_2\Delta \ell_2 - mgy_2 + mgy_1 = \frac{1}{2} m\, v_2^2 - \frac{1}{2} mv_1^2 .$$

The mass m has volume $A_1\Delta \ell_1 = A_2\Delta \ell_2$, so we can substitute $m = \rho A_1\Delta \ell_1 = \rho A_2\Delta \ell_2$. After dividing by $A_1\Delta \ell_1 = A_2\Delta \ell_2$ and rearranging we obtain

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 . \qquad (9\text{-}3)$$

Since points (1) and (2) can be any two points along a tube of flow, the equation (9-3) can be written in form

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant} \qquad (9\text{-}4)$$

at every point in the fluid.

Equations (9-3) or (9-4) are called Bernoulli´s equation.

Example: Calculate the velocity v_1 of a liquid flowing out of a spigot at the bottom of a reservoir in Fig. 9-4.

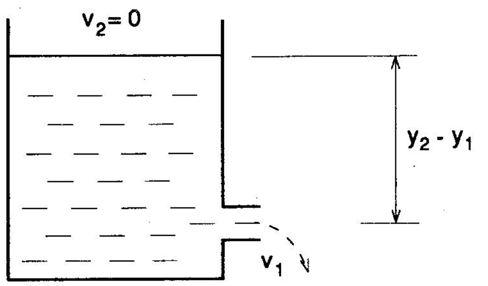

Figure 9-4

Solution: For the using of Eq. (9-3) we choose the spigot as point (1) and the top surface of the liquid as point (2). Assuming the diameter of the reservoir is very large compared to that of the spigot, v_2 will be almost zero. The pressure at both points is the same and it equals to atmospheric pressure, so $P_1 = P_2$.

By eq. (9-3) we can write

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 = \rho g y_2$$

and thus

$$v_1 = \sqrt{2g(y_2 - y_1)} .$$

This result is called Torricelli´s theorem.

9 - 3 Viscosity

Fluids have a certain amount of internal friction called viscosity. It exists in both liquids and gases and is essentially a frictional force between different layers of fluid as they move.

The fluid in contact with the stationary plate remains stationary. This stationary layer of fluid retards the flow of the layer just above it. This layer retards the flow of the next layer above, and so on. Thus the velocity varies linearly from 0 to do as shown in Fig. 9-5. The increase in velocity divided by the distance over which this change is made equals dv/dz and is called the velocity gradient. To move the upper plate requires a force. This force required F is proportional to its area A and to the velocity gradient. For different fluids, the more viscous the fluid, the greater the force must be. Hence the proportionality constant for this equation is defined as the coefficient of viscosity η :

$$F = \eta A \frac{dv}{dz}, \tag{9-5}$$

where the velocity gradient is the rate the velocity changes per unit distance measured perpendicular to the direction of the velocity.

The SI unit of η is N s/m^2 = Pa.s.

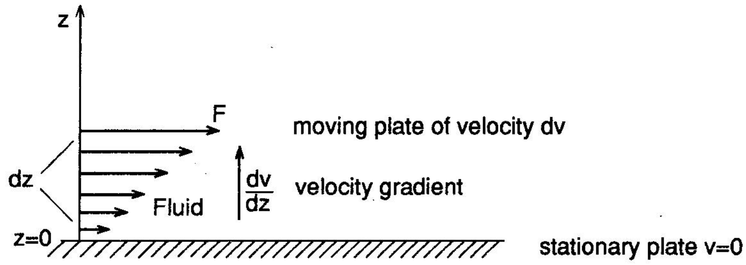

Figure 9 - 5

Notice, the temperature has a strong effect - the viscosity of liquids decreases rapidly as temperature increases.

9 - 4 Laminar Flow in Tubes

Let us consider a fluid undergoing steady laminar flow through a cylindrical tube of inner radius R, as shown in Fig. 9-6.

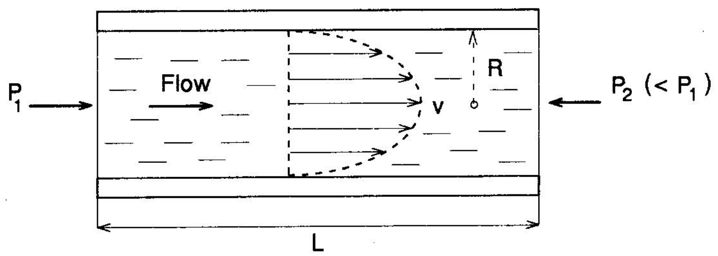

Figure 9 - 6

Since a fluid tends to adhere to the walls of the tube, we can expect that the fluid velocity will be near zero at the walls. We therefore assume the cylindrical layer of fluid next to the wall tube has zero velocity. Each successive layer has only a slightly larger velocity because of the viscous friction with the previous layer. The velocity thus increases with distance from the wall and reaches a maximum at the center of the tube (see Fig. 9-6).

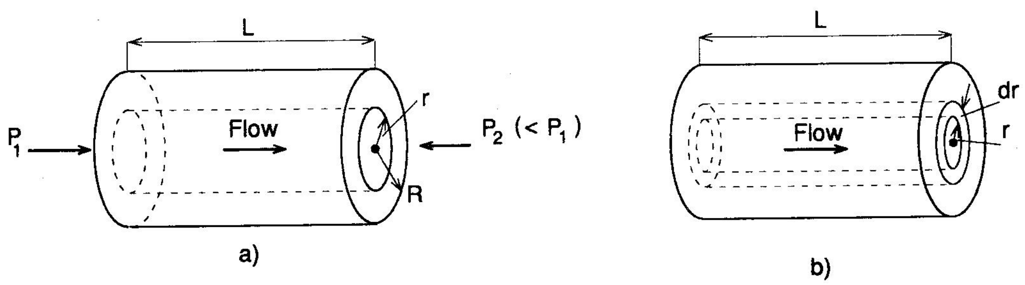

Figure 9 - 7

We first determine v as a function of r by considering a solid cylinder of fluid of radius $r < R$ whose center line is along the center of the tube (see Fig. 9-7a). The force on this cylinder due to the difference in pressure at the ends of the tube is

$$F = (P_1 - P_2)\pi r^2 \qquad (9\text{-}6)$$

The motion of this cylinder of fluid is retarded by the viscous force exerted by the next layer-cylinder of fluid just outside our assumed cylinder. The magnitude of this viscous force is given by Eq. (9-5) where for the area A we must use the area of the sides of the cylinder, $A = 2\pi rL$; so, the viscous force is

$$F = -\eta 2\pi rL \frac{dv}{dr}, \qquad (9\text{-}7)$$

where the minus sign indicates that this force opposes the motion. Since the fluid is undergoing steady flow, there is no acceleration and these two forces (9-6) and (9-7) must balance:

$$(P_1 - P_2)\pi r^2 = -\eta 2\pi rL \frac{dv}{dr}$$

and

$$\frac{dv}{dr} = -\frac{(P_1 - P_2)}{2\eta L} r .$$

By integrating we obtain v as a function of r the distance from the center of the tube, and note that $v = 0$ at $r = R$:

$$\int_0^v dv = -\frac{P_1 - P_2}{2\eta L}\int_R^r r\, dr ,$$

and

$$v = \frac{P_1 - P_2}{4\eta L}(R^2 - r^2) . \qquad (9\text{-}8)$$

We see that the maximum velocity occurs at the center of the tube ($r = 0$). Its magnitude is proportional to the square of the tube radius and is also proportional to the pressure gradient $(P_1 - P_2)/L$.

Now we know v as a function of r and we can determine the total flow rate Q through the tube, where $Q = dV/dt$ is the total volume of fluid passing a cross section of the tube per unit time. Since the velocity v is not constant across the tube, we divide up the cross section of the tube into thin rings of thickness dr, as shown in Fig. 9-7b. We calculate the flow through each of rings and sum over them all to get the total flow rate. The area of the thin ring shown in Fig. 9-7b is

$$dA = 2\pi r\, dr ,$$

where $2\pi r$ is circumference of ring and dr its width. The flow rate through this ring is

$$dQ = v\, dA = \frac{P_1 - P_2}{4\eta L}(R^2 - r^2)2\pi r\, dr .$$

Total flow rate through the tube is

$$Q = \int_{r=0}^{R} dQ = \frac{\pi(P_1 - P_2)}{2\ L}\int_0^R (R^2 r - r^3)\, dr =$$

$$= \frac{\pi(P_1 - P_2)}{2\eta L}\left[R^2\frac{r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{\pi(P_1 - P_2)R^4}{8\eta L} . \qquad (9\text{-}9)$$

This equation is sometimes called Poiseuille's equation. It tells us that the flow rate Q is directly proportional to the prèssure gradient $(P_1 - P_2)/L$ and inversely proportional to the viscosity of the fluid. Q also depends on the fourth power of the tube's radius this means for example, that for the same pressure gradient, if the tube radius is doubled the flow rate is increased by a factor of 16.

10. TEMPERATURE

In this and the next three chapters we will study the intimately related topics of temperature, kinetic theory, heat and thermodynamics. The emphasis in this chapter is on the concept of temperature. We will often consider a particular system, by which we mean a particular object or set of objects;everything else in the universe is called the "environment". In order to describe the state of a particular system - such a gas in a container - we will use the quantities that are more or less detectable by our senses, such as volume, mass, pressure and temperature. The number of macroscopic variables required to describe the state of a system at any time depends on the type of system. To describe the state of a pure gas in a container, for example, we need only three variables, which could be the volume, the pressure and the temperature. These quantities, that can be used to describe the state of the system, are called state variables.

10 - 1 Temperature, Temperature Scales

We begin our examination of heat phenomena with a study of temperature. In everyday life, temperature refers to how hot or cold an object is. Many properties of matter change with temperature. Among these are the volume of a liquid, the length of a rod, the electrical resistance of a wire, the pressure of a gas kept at constant volume, the volume of a gas kept at constant pressure, the color of a lamp filament etc. Any of these properties can be used in the construction of a thermometer that is, in the setting up of a temperature scale. We then define this temperature scale by an assumed continuous monotonic relation between the chosen thermometric property and the temperature as measured on our scale. For example, the thermometric substance may be a liquid in a glass capillary tube and the thermometric property can be the length of the liquid column.

Suppose that we have chosen a thermometric substance. Let us represent by x the thermometric property that we wish to use in setting up a temperature scale. We arbitrarily choose the following linear function of the property x as the temperature T which the appropriate thermometer has:

$$T(x) = \alpha x$$

In this expression α is a constant which we must still evaluate. By choosing this linear form for $T(x)$ we have fixed it so that equal temperature differences correspond to equal changes in x . It is also obvious that two temperatures, measured with the same thermometer, are in the same ratio as their corresponding x's, that is,

$$\frac{T(x_1)}{T(x_2)} = \frac{x_1}{x_2} . \qquad (10\text{-}1)$$