15. ELECTRIC CURRENT

Until the year 1800, the technical development of electricity consisted mainly of producing a static charge by friction. A number of machines had been built that could produce large potentials, but they had little practical value.

In 1800, an event of great practical importance occurred. A. Volta invented the electrical battery, and with it produced the first steady flow of electric charge - that is, a steady electric current.

15 - 1 Current, Current Density, Conservation of Charge

In metals the valence electrons are not attached to individual atoms but are free to move about within the lattice and are called conduction or free electrons. These electrons are in random motion like the molecules of a gas.

If we have a metallic conductor which is connected to a battery, an electric field will be set up at every point within the conductor. This field $\vec{E}$ will act on the electrons and will give them a resultant motion in the direction of $-\vec{E}$. We say that an electric current is established. The average electric current in a conductor is defined as

$$I = \frac{\Delta Q}{\Delta t}, \qquad (15\text{-}1)$$

where ΔQ is the net amount of charge that passes through a cross section of conductor at a given point during the time interval Δt. If the current is not constant in time, then we can define the instantaneous current at any moment as the infinitesimal limit as $\Delta t \rightarrow 0$ so

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}. \qquad (15\text{-}2)$$

Electric current is measured in coulombs per second, this is given a special name ampere (abbr. A).

A more general kind of current involves charge carriers moving around in a three dimensional volume. To describe this we need a concept of a new microscopic quantity - the current density $\vec{j}$. The current density is a vector which is a characteristic of a point inside a conductor rather than a conductor as a whole.

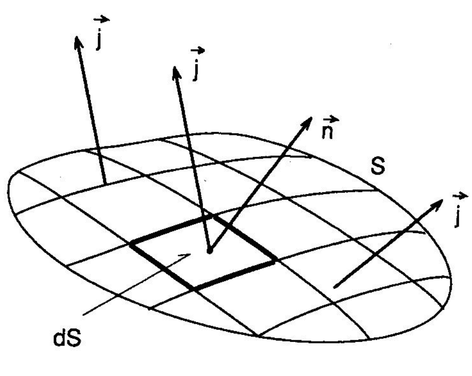

Figure 15 - 1

Thus for the current dI passing through an elementary surface area dS we can write

$$dI = \vec{j}.\vec{dS}, \qquad (15\text{-}3)$$

where

$$\vec{dS} = \vec{n}.dS. \qquad (15\text{-}4)$$

The vector $\vec{n}$ is the unit vector normal to dS, see Fig. 15-1.

The vector $\vec{j}$ at any point is oriented in the direction that a positive charge carrier would move at that point.

The electric current passing through any surface S (see Fig. 15-1) is the integral over

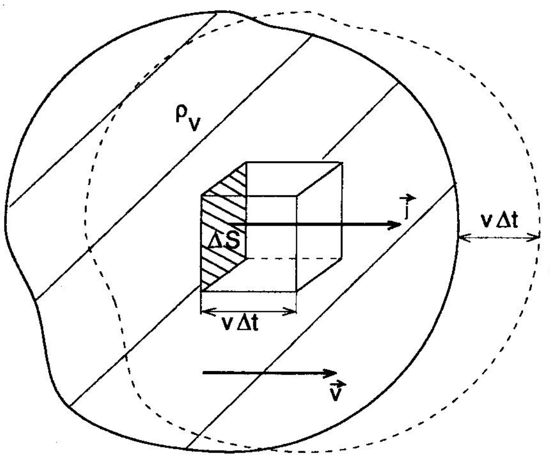

Figure 15 - 2

all the surface in question, so

$$I = \iint_S \vec{j}.\vec{dS} . \qquad (15\text{-}5)$$

The current density is related to the average flow velocity of the charges. Suppose that we have a distribution of charges whose average motion is a drift with the velocity $\vec{v}$. As this distribution passes over a surface element ΔS, the charge ΔQ passing through the surface element in a time Δt is equal to the charge contained in a parallelepiped whose base is ΔS and whose height is $v\Delta t$, as shown in Fig. 15-2.

The volume of the parallelepiped is $\Delta S(v\Delta t)$ which when multiplied by the volume charge density ρ_V will give ΔQ. Thus

$$\Delta Q = \rho_V v \Delta t \Delta S . \qquad (15\text{-}6)$$

The charge per unit time represents the current passing through the surface area ΔS. Coming to the limit $\Delta t \to dt$ and $\Delta S \to dS$ we obtain

$$dI = \frac{dQ}{dt} = \rho_V v\, dS = \rho_V \vec{v}.\vec{dS} . \qquad (15\text{-}7)$$

By comparison of Eq. 15-7 and 15-3 we obtain for the current density

$$\vec{j} = \rho_V \vec{v} . \qquad (15\text{-}8)$$

If the charge distribution consists of individual charges, each with charge e , number of charges per unit volume n and the charges are moving with the mean velocity $\vec{v}$, then the current density is

$$\vec{j} = en \vec{v} . \qquad (15\text{-}9)$$

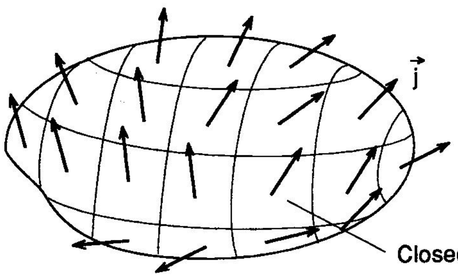

Figure 15 - 3

The current I out of a closed surface S represents the rate at which the charge leaves the volume V enclosed by S. One of the basic laws of physics (see Section 14-1) is that electric charge is indestructible; it is never lost or created. Electric charges can move from place to place but never appear from nowhere. If there is a net current out of closed surface, the amount of charge inside must decrease by the corresponding amount (Fig. 15-3).

We can, therefore, write the law of conservation of charge as

$$\oint\!\!\oint_S \vec{j}.\vec{dS} = -\frac{d}{dt}(Q_{inside}) \qquad (15\text{-}10)$$

The charge inside can be written as a volume integral of the charge density

$$Q_{inside} = \iiint_V \rho_V \, dV .$$

Thus we obtain

$$\oint_S \vec{j}\cdot\vec{dS} = -\frac{d}{dt}\iiint_V \varrho_V \, dV . \qquad (15\text{-}11)$$

Letting the volume in question shrink down around any point (x, y, z) the conservation of charge law can be written

$$\mathrm{div}\,\vec{j} = -\frac{\partial \varrho_V}{\partial t} . \qquad (15\text{-}12)$$

No charge can flow away from a place without diminishing the amount of charge that is there.

The time derivative of the charge density ϱ_V is written as partial derivative since ϱ_V will usually be a function of spatial coordinates as well as time.

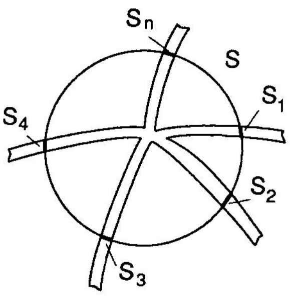

Figure 15 - 4

As an example we use the conservation of charge law to determine the current in the point of network where several conductors meet, see Fig. 15-4.

We shall suppose stationary case, that is the case when a volume charge density ϱ_V is not function of time. Thus we can write Eq. 15-11 as

$$\iint_S \vec{j}\cdot\vec{dS} = 0 . \qquad (15\text{-}13)$$

Conservation of charge law for a closed surface S which contains parts of different conductors (that is S_1, S_2, ..., S_n - see Fig. 15-4) then gives

$$\oint_S \vec{j}\cdot\vec{dS} = \iint_{S_1} \vec{j}\cdot\vec{dS} + \iint_{S_2} \vec{j}\cdot\vec{dS} + \ldots + \iint_{S_n} \vec{j}\cdot\vec{dS} = I_1 + I_2 + \ldots + I_n = 0 \qquad (15\text{-}14)$$

or

$$\sum I_K = 0 . \qquad (15\text{-}15)$$

This equation is a mathematical statement of the Kirchhoff´s first or junction rule: the algebraic sum of the currents into the junction point must be zero. Charges that enter a junction must also leave - none is lost or gained.

15 - 2 Ohm´s Law, Resistance, Resistivity, Conductivity

In order to establish an electric current in a circuit, a difference in potential is required. It was G. S. Ohm (1787 - 1854) who established experimentally that the current in a metal wire is proportional to the potential difference V applied to its ends

$$I = \frac{V}{R} ,$$

where R is called the resistance of the wire. This relation is often written as

$$V = IR \qquad (15\text{-}16)$$

and is referred to as a Ohm´s law, which states that current through a metal conductor is proportional to the applied voltage. That is R is constant and independent on V for metal conductors. However this relation does not apply generally for other substances and devices such as transistors, vacuum tubes etc. Such materials or devices are said to be nonohmic.

The unit for resistance is called the ohm and is abbreviated Ω (Greek capital omega).

All electric devices offer resistance to the flow of current. In many circuits, particulary in electronic devices, the resistors are used to control the amount of current.

It was found experimentally that the resistance R of a uniform metal wire is directly proportional to its length ℓ and inversely proportional to the cross-sectional area S , that is

$$R = \rho \frac{\ell}{S} , \qquad (15\text{-}17)$$

where ρ is called the resistivity and depends on the material used.

The resistivity depends somewhat on purity, heat treatment, temperature and other factors. Silver has the lowest resistivity, but it is expensive. Copper is not far behind so it is clear why the most wires are made of copper.

As it was already said the resistivity of a material depends on temperature. In general, the resistance of metals increases with temperature, that is

$$\rho_T = \rho_0 \left[1 + \alpha (T - T_0)\right] , \qquad (15\text{-}18)$$

where ρ_T is the resistivity at temperature T, ρ_0 is the known resistivity at a standard temperature T_0, and α is the temperature coefficient of resistivity. Note that for semiconductors the coefficient α can be negative.

At low temperatures, the resistivity of certain metals and their compounds or alloys becomes essentially zero. Materials in such a state are said to be superconducting. This phenomenon was first observed by H. K. Onnes in 1911 when he cooled mercury below 4,2 K. He found that at this temperature the resistance of mercury suddenly dropped to zero. In early 1987, a compound of yttrium, barium, copper and oxygen was developed that can be superconducting even above the temperature of liquid nitrogen 77 K. Rapid progress quickly produced materials that superconduct at even higher temperatures.

The reciprocal of the resistivity is called the conductivity σ, that is

$$\sigma = \frac{1}{\rho} . \qquad (15\text{-}19)$$

The conductivity has units of $(\Omega.m)^{-1}$.

Ohm's law can be written in terms of microscopic quantities as follows. The potential difference V applied between the ends of the wire is

$$V = E \ell , \qquad (15\text{-}20)$$

where ℓ is the length of the wire. We suppose that the electric field E within the wire is uniform. The potential difference V is also given by Ohm's law (Eq. 15-16), thus we have

$$E \ell = IR . \qquad (15\text{-}21)$$

Substituting into this equation for resistance Eq. 15-17 and for current (see Eq. 15-5) I = jS we have

$$E \ell = jS \rho \frac{\ell}{S} .$$

Thus we obtain

$$j = \frac{1}{\rho} E \qquad (15\text{-}22)$$

or

$$j = \sigma E . \qquad (15\text{-}23)$$

Eq. 15-22 and Eq. 15-23 can be written in vector form as

$$\vec{j} = \sigma . \vec{E} = \frac{1}{\rho} . \vec{E} . \qquad (15\text{-}24)$$

This expression is a microscopic statement of Ohm's law.

Let us now determine the conductivity σ from a microscopic point of view. When a homogeneous electric field is applied to the electrons in metal it will experience a force $(-e\vec{E})$ on each of them. We suppose the initial velocity of electrons equal to zero, so that we can write

$$-e\vec{E} = m\frac{d\vec{v}}{dt}$$

or

$$\vec{v} = -\frac{e\vec{E}}{m}\bar{t} . \qquad (15\text{-}25)$$

We can see that the velocity of each electron is increased from zero to v_{max} during the time interval $\bar{t}$. This time interval represents mean time between two succeeding collisions. The average velocity, therefore, is

$$\bar{v} = \frac{v_{max}}{2} .$$

Thus we obtain

$$\bar{v} = -\frac{1}{2}\frac{eE}{m}\bar{t} . \qquad (15\text{-}26)$$

From the expression for current density, see Eq. (15-9), we obtain

$$v = \frac{j}{-en} . \qquad (15\text{-}27)$$

Combining Eq. (15-26) and (15-27) we have

$$-\frac{j}{en} = -\frac{1}{2}\frac{eE\bar{t}}{m}E$$

or

$$j = \frac{1}{2}\frac{e^2n\bar{t}}{m}E . \qquad (15\text{-}28)$$

If we compare Eq. (15-28) and Eq. (15-23) we can see

$$\sigma = \frac{e^2n\bar{t}}{2m} . \qquad (15\text{-}29)$$

Equation (15-29) can be taken as a statement that metals obey Ohm's law if we can show that $\bar{t}$ does not depend on the applied electric field E. In this case σ will not depend on E. The quantity $\bar{t}$ depends on the speed distribution of the conduction electrons which is affected only very slightly by the application of even a relatively large electric field. Thus until very strong electric fields $[\sim 10^8 \text{ V.m}^{-1}]$ materials obey Ohm's law.

15 - 3 Electromotive Force

The steady flow of current requires naturally some source of energy capable of maintaining the electric field that drives the charge carriers. There are certain devices such as batteries and electric generators which are able to maintain a potential difference between two points to which they are attached. We call such devices seats of electromotive force (symbol $\mathcal{E}$, abbr. emf). The origin of the emf in a direct-current circuit is some mechanism that transports charge carriers in a direction opposite to that in which the electric fields is trying to move them. We can represent the effect of this mechanism by a certain electric field $\vec{E}^*$.

Imagine we have a simple electric circuit $\mathcal{E}$ containing a seat of emf (see Fig. 15-5).

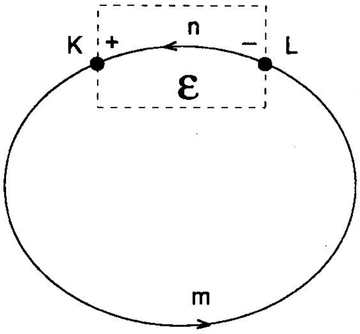

Figure 15 - 5

The flow of charges in this electric circuit can be therefore described by the Ohm's law, that is

$$\vec{j} = \sigma . \vec{E} = \sigma (\vec{E}_S + \vec{E}^*) , \qquad (15\text{-}30)$$

where $\vec{E}_S$ is the electric field which causes the flow of charges outside the seat of emf that is in that part of the circuit which is described as m. This electric field corresponds to the potential difference between the terminals K and L.

Let us divide Eq. (15-30) by σ , then multiply it by an oriented element of the circuit length $\vec{d\ell}$ and finally to integrate it over all closed circuit. Thus we obtain

$$\oint_{(m+n)} \frac{\vec{j}}{\sigma} \vec{d\ell} = \oint_{(m+n)} (\vec{E}_S + \vec{E}^*) \vec{d\ell} . \qquad (15\text{-}31)$$

The integral on the right side can be divided in two parts that is integral over the path m outside the seat of emf and the integral over the path n inside the seat of emf (see Fig. 15-5). Notice, however, that in the part n of the integration path the motion of charges is influenced not only by the electric field $\vec{E}^*$ but also by the electric field $\vec{E}_S$. Thus the motion of charges inside the seat of emf is determined by the electric field $(\vec{E}_S + \vec{E}^*)$. On the contrary outside the seat of emf the electric field $\vec{E}^*$ equals to zero.

We can write right side of Eq. (15-31) as

$$\oint_{(m+n)} \vec{E}.\vec{d\ell} = \oint_{(m+n)} (\vec{E}_S + \vec{E}^*) \vec{d\ell} = \int_K^L (\vec{E}_S + \vec{E}^*) \vec{d\ell} + \int_L^K (\vec{E}_S + \vec{E}^*) \vec{d\ell} =$$

$$= \int_K^L \vec{E}_S.\vec{d\ell} + \int_L^K \vec{E}_S.\vec{d\ell} + \int_L^K \vec{E}^*.\vec{d\ell} = \int_{(m+n)} \vec{E}_S.\vec{d\ell} + \int_L^K \vec{E}^*.\vec{d\ell} . \qquad (15\text{-}32)$$

Integral $\oint_{(m+n)} \vec{E}_S.\vec{d\ell}$ equals to zero as a result of validity of conservation of energy law (see Eq. 14-35).

The integral

$$\int_L^K \vec{E}^*.\vec{d\ell} = \mathcal{E} \qquad (15\text{-}33)$$

is referred to as an electromotive force. Electromotive force may be defined as the work per unit charge passing through the seat of emf.

Similary let us now separate left side of Eq. (15-31) in two parts

$$\oint_{m+n} \frac{\vec{j}}{\sigma} \vec{d\ell} = \int_K^L \frac{\vec{j}}{\sigma} \vec{d\ell} + \int_L^K \frac{\vec{j}}{\sigma} \vec{d\ell} . \qquad (15\text{-}34)$$

Let us imagine a steady current through a homogeneous conductor. In this case vectors $\vec{j}$ and $\vec{d\ell}$ are colinear. We can also write

$$j = \frac{I}{S} \qquad \text{and} \qquad R = \frac{\ell}{\sigma S} ,$$

where ℓ is the length of conductor between points K and L. Thus we obtain

$$\int_K^L \frac{\vec{j}}{\sigma}\, d\vec{\ell} = \int_K^L \frac{j}{\sigma}\, d\ell = \frac{I}{S\sigma}\int_K^L d\ell = I\,\frac{\ell}{\sigma S} = IR \,. \qquad (15\text{-}35)$$

For the second integral in Eq. (15-34) we can write similary

$$\int_L^K \frac{\vec{j}}{\sigma}\, d\vec{\ell} = IR_i \,, \qquad (15\text{-}36)$$

where R_i is called internal resistance.

Substituting Eq. (15-33, Eq. (15-35),Eq. (15-36) into (Eq. 15-31) we obtain

$$\mathcal{E} = I(R + R_i) \,. \qquad (15\text{-}37)$$

This equation represents Ohm´s law rewritten for a closed circuit.

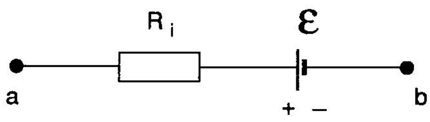

Figure 15 - 6

The internal resistance can be represented as if it were in series with emf, as shown in Fig. 15-6.

The two points a and b in the diagram represent the two terminals of the seat of emf for example battery. What we measure is the terminal voltage V_{ab}. When no current is drawn from the battery the terminal voltage equals the emf. However, when a current I flows from the battery, there is a drop in voltage equal to IR_i. Thus the terminal voltage is

$$V_{ab} = \mathcal{E} - IR_i \,. \qquad (15\text{-}38)$$

For example if 12 V battery has an internal resistance of $0{,}1\,\Omega$, then when 10 A flows from the battery, the terminal voltage is 11 V.

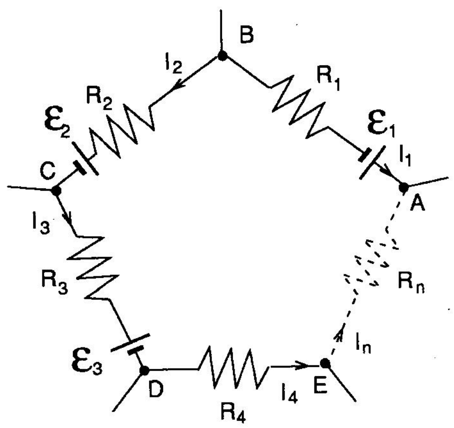

Figure 15 - 7

As an example of the presented results we shall obtain Kirchhoff´s second rule or so called loop theorem. Imagine closed electric circuit which consists from junctions A, B, C, D, E and branches connecting these junctions, see Fig.15-7.

Each branch may contain a resistance R and an electromotive force $\mathcal{E}$.

A convenient starting point is Eq. (15-32) which we can rewrite for the presented electric circuit as

$$\oint \vec{E}.d\vec{\ell} = \oint \vec{E}_S.d\vec{\ell} + \oint \vec{E}^*\, d\vec{\ell} \qquad (15\text{-}39)$$

or with respect to the fact that $\oint \vec{E}_S.d\vec{\ell} = 0$ we have

$$\oint \vec{E}.d\vec{\ell} = \oint \vec{E}^*.d\vec{\ell} \,. \qquad (15\text{-}40)$$

We can write the integrals on both sides of Eq. (15-40) as a sum of integrals along different branches of the closed circuit so that we have

$$\oint \vec{E}.d\vec{\ell} = \int_A^B \vec{E}.d\vec{\ell} + \int_B^C \vec{E}.d\vec{\ell} + \ldots + \int^A \vec{E}.d\vec{\ell} \,, \qquad (15\text{-}41)$$

$$\oint \vec{E}^*.d\vec{\ell} = \int_A^B \vec{E}^*.d\vec{\ell} + \int_B^C \vec{E}^*.d\vec{\ell} + \ldots + \int^A \vec{E}^*.d\vec{\ell} \,. \qquad (15\text{-}42)$$

The integrals on the right side of Eq. (15-41) equal to the products of resistances and currents in different branches so that we have

$$\oint \vec{E}.d\vec{\ell} = R_1I_1 + R_2I_2 + \ldots + R_nI_n = \sum_{k=1}^{n} R_kI_k . \quad (15\text{-}43)$$

The integrals on the right side of Eq. (15-42) equal to the electromotive forces in different branches so that we have

$$\oint \vec{E^*}.d\vec{\ell} = \mathcal{E}_1 + \mathcal{E}_2 + \ldots + \mathcal{E}_n = \sum_{k=1}^{n} \mathcal{E}_k . \quad (15\text{-}44)$$

Substituting Eq. (15-43) and Eq. (15-44) in Eq. (15-40) we have

$$\sum_{k=1}^{n} \mathcal{E}_k = \sum_{k=1}^{n} R_kI_k$$

or

$$\sum_{k=1}^{n} \mathcal{E}_k - \sum_{k=1}^{n} R_kI_k = 0 , \quad (15\text{-}45)$$

where n is number of branches. Eq. (15-45) is a mathematical statement of the Kirchhoff's second rule: the algebraic sum of the changes in potential around any closed path of the circuit must be zero. Note that this rule is based on the conservation of energy law.

Kirchhoff's first and second rules are used together to find for example currents when the emf and the resistances in multiloop circuits are given.

15 - 4 Energy Transfers in an Electric Circuit

Electric energy is useful to us because it can be easily transformed into other forms of energy.

Figure 15-8 shows a circuit consisting of a battery B connected to a "black box". A steady current I exists in the connecting wires, a steady potential difference V exists between the terminals a and b. The box might contain a resistor, a motor or a storage battery among other things.

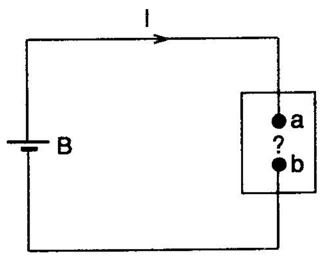

Figure 15 - 8

To find the energy transformed by such a "black box" we use the fact that the energy dU transformed when an infinitesimal charge dq moves through a potential difference V (see Eqs. (14-37) and (14-38)) is

$$dU = dq\, V . \quad (15\text{-}46)$$

If dt is the time required for an amount of charge dq to move through V, the power P, which is the rate of energy transformed is

$$P = \frac{dU}{dt} = \frac{dq}{dt} V . \quad (15\text{-}47)$$

The charge that flows per second is simply the current. Thus we have

$$P = IV . \quad (15\text{-}48)$$

This is a general relation which gives us the instantaneous power of any device, where I is the current passing through it and V is the potential difference across it.

The rate of energy transformation in a resistance R can be written, by combining Eq. (15-48) and Eq, (15-16) as

$$P = I(IR) = I^2R = \left(\frac{V}{R}\right)V = \frac{V^2}{R} . \quad (15\text{-}49)$$

Equation (15-49) is known as Joule's law. Note that Eq. (15-48) applies to electrical energy transfers of all kinds and Eq. (15-49) applies only to the transfer of electrical energy to thermal energy in a resistor.

The SI unit of electric power is the same as for any kind of power, namely the watt

$$1\ W = \frac{1\ J}{1\ s} .$$

Let us now obtain Joule's law from a microscopic point of view. Imagine we have a conductor, see Fig. 15-9, which is connected to a battery. The electric field $\vec{E}$ acts on the charge carriers so that these charges will travel with a constant drift speed $\vec{v}$. The electric potential energy that they lose is transformed to the resistor as thermal energy. On a microscopic scale we can understand this in that collisions between the charge carriers and the lattice increase the amplitude of the thermal vibrations of the lattice. On the macroscopic scale this corresponds to a temperature increase.

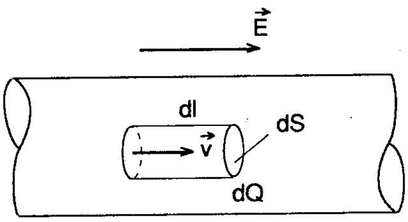

Figure 15 - 9

In this conductor we choose an elementary volume dV which has a cross section dS and the length $d\ell$. We define power density as

$$p = \frac{dP}{dV} . \qquad (15\text{-}50)$$

The power P can be expressed as $P = \vec{F}.\vec{v}$ (see Eq. 4-5). As far as the force $\vec{F} = Q.\vec{E}$ we can write

$$p = \frac{d}{dV}(\vec{F}.\vec{v}) = \frac{d}{dV}(Q.\vec{E}.\vec{v}) . \qquad (15\text{-}51)$$

We consider a stationary case, that is vectors $\vec{E}$ and $\vec{v}$ are constant in volume in question. So we have

$$p = \vec{E}.\vec{v}\,\frac{dQ}{dV} = \vec{E}.\vec{v}\,\varrho_V , \qquad (15\text{-}52)$$

where ϱ_V is the volume charge density (ϱ_V = en).

Equation (15-9) states that $\vec{j} = en.\vec{v}$, so we can write Eq. (15-52) as

$$p = \vec{E}.e.n.\vec{v} = \vec{E}.\vec{j} . \qquad (15\text{-}53)$$

This equation is a mathematical statement of the Joule's law from the microscopic point o view.

The SI unit of power density is $W.m^{-3}$ (see Eq. 15-50).

From Eq. (15-47) it is possible to determine the total energy E used by any device during a time t as

$$E = \int_0^t P\,dt \qquad [J] \qquad (15\text{-}54)$$

15 - 5 Conduction of Electricity in Gases

In their normal state gases are electrical insulators. This is due to the fact that they contain no free charged particles but only neutral atoms or molecules. If, however, electric fields of sufficient intensity are applied to them they become conductive and the complex phenomena which then occur are called gas discharges;they are due to the appearance of free electrons and ions.

The result of a gas discharge is therefore to produce an ionized gas. Probably the most important reaction of a gas discharge is ionization. When an electric field is placed across a gas, the electrons that may be in the gas volume are accelerated by that field. These electrons are continually colliding with the neutral gas atoms. After many such collisions, the electric field will have accelerated some of these electrons to energies greater than so called ionization potential. This potential corresponds to the energy when the collision between the energetic electron and the neutral atom can cause an electron to be ejected from the atom. This will create a positive ion and a new free electron. This proces is called the ionization.

The reversible process which occurs when a positive ion collides with the electron so that the neutral atom is obtained is called recombination.

The neutral gas can be also ionized by a high temperature (thermal ionization), high pressure, by irradiation with X-rays etc. The most important role in practice plays, however, the ionization in an electric field, so we shall discuss gas discharges.

Gas discharges can take place over a very wide range of gas pressure and carry currents ranging from scarcely measurable values to 10^7 A and more. They may be steady state processes or transient or very short duration. The behaviour of a gas discharge is in general influenced by the properties of the electrical circuit of which it forms a part.

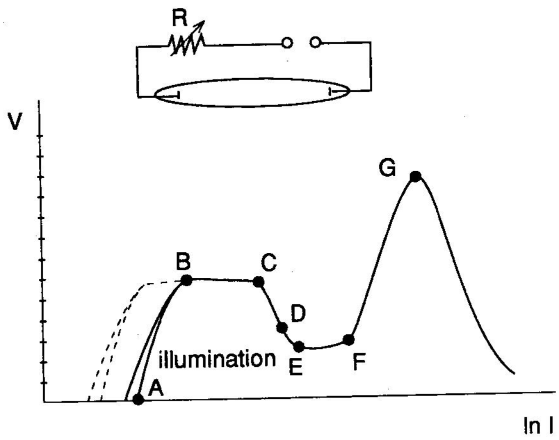

Figure 15 - 10

Consider a cylindrical glass tube with two plane electrodes at its ends filled with a gas at low pressure (about 10^3 Pa). A d.c. potential is applied aross the electrodes from a source in series with a variable resistor to control the current flowing through the circuit, see Fig. 15-10.

As the potential V across the electrodes is increased very slowly we observe first very small currents occuring in random bursts. A somewhat larger steady current would be observed if the cathode were illuminated with ultra violet light, the current becoming larger with increased intensity of illumination. Removal of the light causes the current to revert to random bursts. This is shown in Fig. 15-10, region AB.

If now the potential V is further increased, the current begins to increase rapidly. The current is determined by the resistance R in the circuit and does not change if the light shining on the cathode is removed: it is independent of any external source of ionization and this state of affairs constitutes a self sustained discharges. The discharges of this kind are usually called Townsend or Dark discharges (region BC, Fig. 15-10).

If the current is allowed to increase further the discharge becomes faintly visible with light and dark spaces arranged along the tube in a characteristic manner and the potential across the electrodes drops considerably, until it reaches a constant value. This region is shown as CDE in Fig. 15-10 and a glow discharge is said to have been established.

If the current is allowed to increase further the voltage remains constant even when the current varies over 2 or 3 orders to magnitude. This is a region of a normal glow discharge (region EF in Fig. 15-10). If the current is still further increased the potential across the discharge rises considerably: this is an abnormal discharge (region FG, in Fig. 15-10). Still further increase in current causes the potential to rise to a maximum and at large currents to fall to a very low value. We have then an arc discharge. Here becomes important notably heating of the gas.

There are, however, other types of discharges which take place in atmosferic pressure. First of all there is a corona discharge which takes place in a non-uniform electric field. The breakdown voltage of such a discharge strongly depends on the geometry of the electrodes and their polarity. The corona discharge is of particular importance in high-voltage engineering where non-uniform fields are unavoidable.

Transient discharges, or sparks, can occur over a wide range of current and pressure. If they are initiated, as is usual, by breakdown, then the discharge will last for a time which depends on the source of energy. A.c. discharges at low frequencies may be classified much as the d.c. cases, but increasing frequency changes the behaviour radically and distinction becomes less clear.

The existence of gas discharges in nature is familiar through lightning and through the Aurora Borealis which occurs in outer atmosphere at very low pressure.

The result of a gas discharge is to produce an ionized gas containing, for instance, n_e electrons, n_i positive ions and n_0 neutral molecules per m^3. Such an ionized gas is called plasma. From a strictly scientific standpoint, however, plasmas form an extremely interesting field of study. It has often been said that plasma constitutes the fourth state of matter. The interactions among its constituents are due to the long range of Coulomb forces.

The results of plasma physics are used in a great number of application. Let us mention the following - illumination by sodium lamps, noise generators for radio-electrical measurements, plasma technologies, plasma displays, plasma jets; ionic rocket propulsion, re-entry of rockets into the earth's atmosphere etc. However, the most important goal which waites for the solution by the plasma physicists is the harnessing of nuclear fusion energy.

15 - 6 Conduction of Electricity in Liquids

In 1833, Michel Faraday observed that pure water was almost a perfect insulator, whereas aqueous solutions of certain substances were electrically conducting. If two electrodes of some metal such as platinum are dipped into a distilled water, and one electrode is connected to the positive terminal of a d.c. source, the other to the negative, practically no current is observed.

However a small amount of, for example, sodium chloride ($NaCl$) when dissolved in the water, provides a solution whose resistance is sufficiently low for the current to be appearable. The resistance of the solution depends markedly on the concentration and upon the temperature.

A solution which conducts an electric current is called an electrolyte. The conduction phenomenon which is attended by secondary chemical effects is called electrolysis. The vessel which holds the electrolyte and the electrodes is called an electrolytic cell. The most striking effects that accompany electrolysis are the chemical reactions that take place at the electrodes. Thus, with platinum electrodes in dilute sulfuric acid, hydrogen is formed and liberated as bubbles of gas at the negative electrode, whereas oxygen is formed and liberated as bubbles of gas at the positive electrode. These gases may be collected, dried and weighed, and the first quantitative measurements made by Faraday consisted in noting the substances that formed at both electrodes and measuring their respective masses, after a known current had existed for a known time.

Imagine three electrolytic cells, all equipped with platinum electrodes (to avoid complications that might result if the electrodes themselves were chemically active) but with different aqueous solutions as electrolytes, as shown in Fig.15-11.

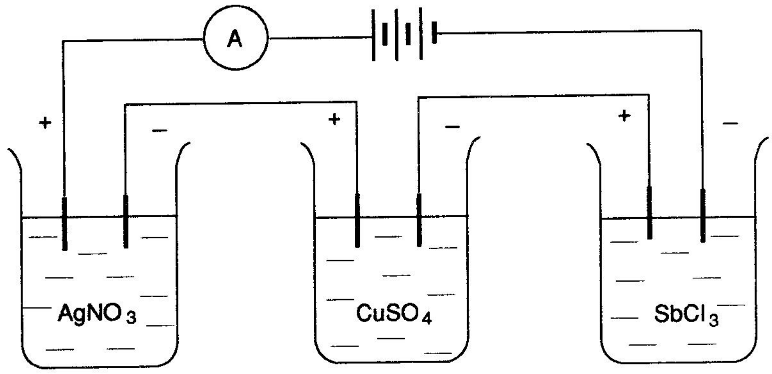

Figure 15 - 11

Connecting these cells in series ensures that the current is the same in all cells. Experiment shows that silver is deposited on the negative electrode of the left-hand cell, copper on the negative electrode of the middle cell, and antimony on the negative electrode of the right-hand cell.

Let us suppose that a current has been maintained for such a time that the electric charge 96 519C has been transferred through each cell. When the three electrodes are removed and the mass of material on each is weighted the following results are obtained:

Silver : 108 g $= \frac{108}{1}$ g

Copper : 31,8 g $= \frac{63,6}{2}$ g

Antimony: 40,7 g $= \frac{122}{3}$ g

These fractions have a simple interpretation. The numerators are the atomic weights of the respective elements, and the denominators are their respective valences. A mass equal to the atomic weight divided by the valence is called an equivalent weight. The electric charge 96 510 is called one faraday, F. Faradays law of electrolysis is therefore: the number of gram-equivalent weights of a substance deposited, liberated, dissolved, or reacted at an electrode is equal to the number of faradays of electricity transferred through the electrolyte.

If a current I is maintained for a time t , then the number of faradays of electricity transferred is

$$\frac{I\,t}{F}.$$

If a mass m of a substance of atomic weight M and of valence j is deposited, then the number of gram-equivalents is

$$\frac{m\,j}{M} .$$

Faraday's law may therefore be written as

$$\frac{m\,j}{M} = \frac{I\,t}{F} . \qquad (15\text{-}55)$$

The electrolysis is widely used in practice for example for refining of metals, in galvanic cells etc.

16. MAGNETIC FIELD

The science of magnetism grew from the observation that certain "stones" (magnetit) would attract bits of iron. The word magnetism comes from the district of Magnesia in Asia Minor, which is one of the places at which the stones were found. Today it is clear that magnetism and electricity are closely related. This relation was not discovered, however, untill 1820 when H. Ch. Oersted discovered that a current in a wire can also produce magnetic effects, namely that it can change the orientation of a compass needle. A compass needle placed near a straight section of current - carrying wire align itself so it is tangent to a circle drawn around the wire. Oersted had therefore found a connection between electricity - that is between movement of charges and magnetism.

16 - 1 Magnetic Field, Definition of $\vec{B}$

We define the space around a magnet or a current - carrying conductor as the site of a magnetic field, just as we defined the space near a charged rod as the site of an electric field.

Let us define the basic magnetic field vector $\vec{B}$, which is called the magnetic induction. For this let us place the test charge q in the electric and magnetic field. The force on this electric charge depends not only where it is, but also on how fast it is moving. Every point in space is characterized by two vector quantities which determine the force on any charge. First, there is the electric force which gives a force component independent of the motion of the charge.

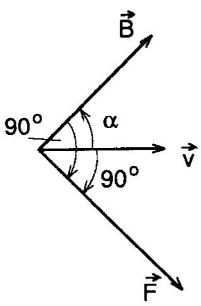

Figure 16 - 1

We describe it by the electric field $\vec{E}$. Second, there is an additional force component, called the magnetic force, which depends on the velocity of the charge. This magnetic force has a strange directional character: at every instant the force is always at right angles to the velocity vector (see Fig. 16-1).

It is possible to describe all of this behaviour by defining magnetic induction vector $\vec{B}$, which specifies both the unique direction in space and the constant of proportionality with the velocity, and to write the magnetic force as $q(\vec{v} \times \vec{B})$. The total electromagnetic force on a charge can be written as

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) . \qquad (16\text{-}1)$$