Physics 1

Electrostatics

Ing. Jaroslav Jíra, CSc.

Electric Charge is the basis of electricity.

The charge on an atom is determined by the subatomic particles an atom consists of.

Elementary charge *e* = 1.602 x 10-19 C

Proton - has a positive charge (+e) and is located in the nucleus.

Neutron - has no charge (is neutral) and is also located in the nucleus as it fills in the spaces between the protons.

Electron - has a negative charge (-e) and is located outside of the nucleus in an electron cloud around the atom.

Numbers Concernig an Atom

Diameter of a nucleus -1 fm = 10^{-15} m (the smallest nuclei) Diameter of an atom $- 0.1$ nm $= 10^{-10}$ m (hydrogen atom) Mass of a proton or neutron – m_{p} = 1.67 x 10⁻²⁷ kg Mass of an electron – m_e = 9.1 x 10⁻³¹ kg Charge of a proton – Q_p = +*e*= +1.602 x 10⁻¹⁹ C Charge of an electron – Q_e = - e = -1.602 x 10⁻¹⁹ C Frequency of electron revolution $-f_e = 6.5 \times 10^{15}$ Hz (first orbit)

Mass of a proton is 1835 times larger than that of an electron. The similar ratio, like masses of a melon and cherry.

Some Facts about Electric Charge

Charge is denoted as *Q* or *q*.

Charge has a fundamental unit of a Coulomb (C).

Charges can exist only in multiples of *e* (elementary charge).

One Coulomb is quite large unit:

- A glass rod rubbed with piece of silk acquires a charge of $10 \mu C$
- A filtration capacitor in a DC source stores a charge of 1 mC
- An average lightning bolt carries a charge of 15 C

Charge cannot be created or destroyed – charge conservation principle.

Atoms usually have as many electrons as protons, so the atom has a zero net charge (is electrically neutral).

An atom which loses some electrons becomes a positive ion.

An atom which acquires excessive electrons becomes a negative ion.

Conservation of Charge

Some materials tend to give up electrons and become positively charged and some materials tend to attract electrons and become negatively charged.

If we try to rub the glass rod with the silk cloth we find that positive charge appears on the rod. At the same time an equal amount of negative charge appears on the silk cloth, so that the net rod-cloth charge is actually zero. This means that rubbing does not create charge but only transfers it from one body to the other.

Charge conservation can be expressed by:

Net charge before = Net charge after

Coulomb's Law

An electric charge exerts a force on the other charge. Ch. A. Coulomb found that the force is proportional to the product of both charges and inversely proportional to the square of their distance.

where ${\vec F}_{12}$ is the vector force on charge ${\sf Q}_1$ due to \emph{Q}_{2} and \vec{r}_{21} ⁰ is a unit vector pointing from \emph{Q}_{2} to Q_1 .

A simplified notation of the Coulomb's law is sometimes being used.

$$
\vec{F}_{12} = k \frac{Q_1 Q_2}{r_{21}^2} \vec{r}_{21}^0
$$

 $0 - \frac{I_{21}}{I_{21}}$

21

 \vec{r}_{21} $\vec{r}_{21}^0 = \frac{r_{21}}{r_{21}}$

 r_{21}

 $21 -$

Coulomb's Law

Coulomb's Law
 $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r_{21}^2} \vec{r}_{21}^0$

Coulomb's law yields an important finding. If the

ve the same signs, then their product is positive

away from the Q_2 – it is repulsive. If the ch **Coulomb's Law**
 $\frac{1}{\pi \varepsilon_0} \frac{Q_1 Q_2}{r_{21}^2} \frac{r_{21}^2}{r_{21}^2}$
 $\leftarrow \frac{\overline{F}_1}{\sqrt{\varepsilon_1}} \frac{Q_1}{r_{21}}$
 $\frac{Q_1}{\sqrt{\varepsilon_2}} \frac{Q_2}{r_{21}}$
 $\frac{Q_2}{\sqrt{\varepsilon_1}}$

So law yields an important finding. If the charges Q_1 an The Coulomb's law yields an important finding. If the charges *Q¹* and Q_2 have the same signs, then their product is positive and the force \vec{F}_{12} points away from the Q_2 – it is repulsive. If the charges have different signs, then their product is negative and the force ${\vec F}_{12}$ points towards Q_2 – it is attractive.

We can summarize it by saying that like charges repel and unlike charges attract.

Electric Field

The electric field is a vector field; it consists of a distribution of vectors, one for each point in the region around a charged object. To define the electric field at some point P near the charged object, we place a positive test charge q_0 at the point in space that is to be examined and we measure the force \vec{F} acting on the charge.

The electric field is defined as

In order to visualize the electric field we draw a series of lines called electric field lines or force lines.

The electric field of a charged particle points radially away from the (+) charge or radially towards the (-) charge.

Electric Field

Basic properties of the field lines:

- 1. Field lines emanate from a point charge symmetrically in all directions.
- 2. Field lines originate on positive charges and terminate on negative ones. They cannot stop in the midair, but they can extend to infinity.
- 3. Field lines can never cross.
- 4. The tangent to a force line gives the direction of \vec{E} at that point.
- 5. The density of force lines corresponds to the magnitude of \vec{E} .

Electric field from a point charge Q

Let a test charge q_0 be placed at a distance *r* from a point charge *Q*. The magnitude of the force acting on q_o is from the Coulomb's law: Fic field from a point

ie placed at a distance

ie Q. The magnitude of

is from the Coulomb's

lectric field at the

arge is given by:

tric field from a group

to each charge at the gi

rrately calculated field:

rratel **Electric field from a point charge Q**

ge q_0 be placed at a distance

charge Q. The magnitude of
 $F = \frac{1}{4\pi \varepsilon_0} \frac{Qq_0}{r^2}$

a of electric field at the

rest charge is given by:
 $E = \frac{F}{q_0} = \frac{1}{4\pi \varepsilon_0} \frac{Q$

The magnitude of electric field at the position of the test charge is given by:

$$
F = \frac{1}{4\pi\varepsilon_0} \frac{Qq_0}{r^2}
$$

harge Q

\n
$$
F = \frac{1}{4\pi\varepsilon_0} \frac{Qq_0}{r^2}
$$
\n
$$
E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}
$$
\npoint charges is to be

\non point

\nvectorially to find the

In case that an electric field from a group of point charges is to be examined we can: int charge Q
 e^e

of $F = \frac{1}{4\pi\varepsilon_e}$
 $E = \frac{F}{q_0}$

up of point charge a given point

elds vectorially
 $\sum_{i=1}^n \vec{E}_i$ **ectric field from a point charge Q**
 q_o be placed at a distance

arge Q. The magnitude of
 $r = \frac{1}{4\pi \varepsilon_0} \frac{Qq_0}{r^2}$

of electric field at the

t charge is given by:
 $E = \frac{F}{q_0} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$

elect

- 1. Calculate $\overrightarrow{E_n}$ due to each charge at the given point
- 2. Add these separately calculated fields vectorially to find the resultant field

$$
\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n = \sum_{i=1}^n \vec{E}_i
$$

Example - Electric field from two charges (a dipole)

Determine electric field at the point P due to charges $Q_1 = +12 \times 10^{-9}$ C, $Q_2 = -12 \times 10^{-9}$ C.

Firstly we calculate magnitudes and then *x* and y components of both vectors \vec{E}_1 , \vec{E}_2 due to charges Q_1 , Q_2 .

$$
E_1 = E_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{12 \times 10^{-9}}{0.1^2} = 10790 V/m
$$

Magnitudes E_1 and E_2 are equal due to the

Example - Electric field from two charges (a dipole)
\nDetermine electric field at the point P due to
\ncharges
$$
Q_f=+12\times10^{-9}
$$
 C, $Q_2= -12\times10^{-9}$ C.
\nFirstly we calculate magnitudes and then x
\nand y components of both vectors \vec{E}_1, \vec{E}_2
\ndue to charges Q_1, Q_2 .
\n
$$
E_1 = E_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{12\times10^{-9}}{0.1^2} = 10790V/m
$$
\n\nMagnitudes E_1 and E_2 are equal due to the
\nsame distance and charge magnitude.
\n
$$
E_{1x} = E_1 \cos \alpha = 10790 \cdot \cos 60^\circ = 5395V/m; \quad E_{2x} = E_2 \cos \alpha = 5395V/m
$$
\n
$$
E_{1y} = E_1 \sin \alpha = 10790 \cdot \sin 60^\circ = 9344V/m; \quad E_{2y} = E_2 \sin \alpha = 9344/m
$$
\n
$$
E_x = E_{1x} + E_{2x} = \frac{10790V/m}{1000V/m; \quad E_y = E_{1y} - E_{2y} = \frac{0V/m}{1000V/m}}
$$
\n**Answer:** The magnitude of resulting electric field is 10790 V/m and
\nthe direction is parallel to the horizontal axis, positive direction.

the direction is parallel to the horizontal axis, positive direction.

Electric field for the continuous charge distribution

If the charge distribution is a continuous one, the field it sets up at a point P can be calculated by dividing the charge into infinitesimal elements *dq*. Each of these elements produces an electric field d **lectric field for the continuous charge distribution**

harge distribution is a continuous one, the field it sets up at a
 r^2 can be calculated by dividing the charge into infinitesimal

ts dq . Each of these elements Electric field for the continuous charge distribution
If the charge distribution is a continuous one, the field it sets up at a
point P can be calculated by dividing the charge into infinitesimal
elements dq . Each of the **ribution**
 $\frac{d\vec{r}}{dt}$ is sets up at a
 $\frac{d\vec{r}}{dt}$ arge dq and
 $\frac{d\vec{r}}{dt} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\frac{dq}{dS} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$
 $\frac{\Delta q}{\Delta V} = \frac{dq}{dV} \left[\frac{C}{m^3} \right]$ **ibution**
sets up at a
infinitesimal
cifield d \vec{E}
arge dq and
 $=\int d\vec{E}$
 $\frac{q}{l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\frac{q}{s} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$
 $\frac{\Delta q}{N} = \frac{dq}{dV} \left[\frac{C}{m^3} \right]$ distribution
field it sets up at a
e into infinitesimal
electric field d \vec{E}
the charge dq and
 $\vec{E} = \int d\vec{E}$
 $\lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$ tribution

it sets up at a

to infinitesimal

ric field d \vec{E}

charge dq and
 $\vec{E} = \int d\vec{E}$
 $\frac{\Delta q}{\Delta l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\frac{\Delta q}{\Delta S} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$
 $\frac{\Delta q}{\Delta V} = \frac{dq}{dV} \left[\frac{C}{m^3} \right]$ ge distribution

e field it sets up at a

rge into infinitesimal

n electric field d \vec{E}

n the charge dq and

n. $\vec{E} = \int d\vec{E}$
 $= \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $= \lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S} = \frac{dq}{dS} \left[\frac{C$ **ibution**
sets up at a
infinitesimal
dield d \vec{E}
arge dq and
 $=\int d\vec{E}$
 $\frac{d\vec{E}}{dt} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\frac{q}{S} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$ **ibution**
sets up at a
infinitesimal
: field d \vec{E}
arge dq and
 $=\int d\vec{E}$
 $\frac{q}{l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\frac{q}{S} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$
 $\frac{\Delta q}{dV} = \frac{dq}{dV} \left[\frac{C}{m^3} \right]$ **distribution**
field it sets up at a
e into infinitesimal
electric field d \vec{E}
the charge dq and
 $\vec{E} = \int d\vec{E}$
 $\lim_{\Delta t \to 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$
 $\lim_{\Delta V \to 0$ tribution
it sets up at a
io infinitesimal
ic field d \vec{E}
harge dq and
 $\vec{E} = \int d\vec{E}$
 $\frac{\Delta q}{\Delta l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\frac{\Delta q}{\Delta S} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$
 $\frac{\Delta q}{\Delta V} = \frac{dq}{dV} \left[\frac{C}{m^3} \right]$ ge distribution

e field it sets up at a

rge into infinitesimal

n electric field d \vec{E}

n the charge dq and

n. $\vec{E} = \int d\vec{E}$
 $= \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $= \lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S} = \frac{dq}{dS} \left[\frac{C$ **bution**
sets up at a
infinitesimal
field d \vec{E}
arge dq and
 $\int d\vec{E}$
 $\vec{E} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\frac{q}{dS} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$
 $\frac{q}{dV} = \frac{dq}{dV} \left[\frac{C}{m^3} \right]$ **bution**
sets up at a
infinitesimal
field d \vec{E}
arge dq and
 \vec{E}
 $\frac{d\vec{E}}{d\vec{E}}$
 $\frac{d\vec{E}}{d\vec{E}}$
 $\frac{l}{m^2}$
 $\frac{dq}{dS}$ $\left[\frac{C}{m^2}\right]$
 $\frac{q}{V} = \frac{dq}{dV}$ $\left[\frac{C}{m^3}\right]$ **distribution**
field it sets up at a
e into infinitesimal
electric field d \vec{E}
the charge dq and
 $\vec{E} = \int d\vec{E}$
 $\lim_{\Delta t \to 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$
 $\lim_{\Delta V \to 0$ ribution

t sets up at a

c field d \vec{E}

harge dq and
 $=\int d\vec{E}$
 $\frac{q}{\Delta l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $\frac{\Delta q}{\Delta S} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]$
 $\frac{\Delta q}{\Delta V} = \frac{dq}{dV} \left[\frac{C}{m^3} \right]$ **ge distribution**

a field it sets up at a

nge into infinitesimal

n electric field d \vec{E}

n the charge dq and

n. $\vec{E} = \int d\vec{E}$
 $= \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl} \left[\frac{C}{m} \right]$
 $= \lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S} = \frac{dq}{dS} \left[\frac$

2 the noint $0¹$ and $1¹$ 1 dq where r is the $4\pi\varepsilon_{_0}$ r^2 $\;$ the point P $dE = \frac{1}{t} \frac{dq}{r^2}$ where *r* is the dis where *r* is the distance between the charge *dq* and the point P.

 $\lim \frac{\Delta q}{\Delta t} = \frac{uq}{u}$ $\Big|$ $\frac{c}{u}$

The charge can be distributed over a long wire. In this case we talk about the linear charge density τ. $\Delta l \to 0$ Δl and $dl \lfloor n$

The charge can be distributed over a plane. In this case we talk about the surface charge density σ.

The charge can be distributed over a volume. In this case we talk about the volume charge density ρ.

$$
\sigma = \lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S} = \frac{dq}{dS} \left[\frac{C}{m^2} \right]
$$

 $l\rightarrow 0$ Λl

 $\tau = 11m \rightarrow - = -$

$$
\rho = \lim_{\Delta V \to 0} \frac{\Delta q}{\Delta V} = \frac{dq}{dV} \left[\frac{C}{m^3} \right]
$$

An Electric Field of a Dipole

A dipole is represented by two charges of the same magnitude and different signs separated by a distance 2a. We will examine electric field on the vertical axis in the middle between the charges. Using principle of superposition: **ield of a Dipole**
arges of the
s separated
ine electric
dle between
erposition:
 Q
 $\frac{Q}{(2+r^2)}$
nsate each
 $\frac{a}{\sqrt{a^2+r^2}}$; $E = -\frac{1}{4\pi}$ **An Electric Field of a Dij**

A dipole is represented by two charges of the

same magnitude and different signs separated

by a distance 2a. We will examine electric

field on the vertical axis in the middle between

the

$$
\vec{E} = \vec{E}_{+} + \vec{E}_{-}; \quad E_{+} = E_{-} = \frac{1}{4\pi\varepsilon_{0}} \frac{Q}{(a^{2} + r^{2})}
$$

The vertical components compensate each other, so the resulting field is

field on the vertical axis in the middle between
\nthe charges. Using principle of superposition:
\n
$$
\vec{E} = \vec{E}_+ + \vec{E}_-, \quad E_+ = E_- = \frac{1}{4\pi\varepsilon_0} \frac{Q}{(a^2 + r^2)}
$$

\nThe vertical components compensate each
\nother, so the resulting field is
\n $E = 2E_+ \cos \alpha$, where $\cos \alpha = \frac{a}{\sqrt{a^2 + r^2}}$; $E = \frac{1}{4\pi\varepsilon_0} \frac{2aQ}{(a^2 + r^2)^{3/2}}$
\nConsidering $r >> a$ we can simplify
\nThe **dipole moment** is
\n
$$
\frac{E = \frac{1}{4\pi\varepsilon_0} \frac{2aQ}{r^3}}{P = 2aQ}
$$

Considering *r* >>*a* we can simplify

$$
E = \frac{1}{4\pi\varepsilon_0} \frac{2aQ}{r^3}
$$

A Dipole in an Electric Field

The dipole moment can be also considered a vector of magnitude 2*aQ* pointing from the negative charge to the positive one. **A Dipole in an Electric Field**

The dipole moment can be also considered a

vector of magnitude 2aQ pointing from the

negative charge to the positive one.

If a dipole is placed in external electric

field \vec{E} at an

If a dipole is placed in external electric field \vec{E} at an angle α , there are two

$$
\vec{F} = Q \cdot \vec{E}
$$

 \vec{p}

The net force is zero but there is a torque τ about the axis through 0.

After substituting $p = 2aQ$ we obtain

The equation can be also written in vector notation

$$
\tau = pE\sin\alpha
$$

$$
\left| \vec{\tau} = \vec{p} \times \vec{E} \right|
$$

A Dipole in an Electric Field

Work *W* must be done to change the orientation of the dipole in an external field. This work is stored as **potential energy** *U*. The reference angle for zero potential energy is α=90°. **A Dipole in a**
must be done to char
of the dipole in an ϵ
work is stored as **pc**
The reference angle f
nergy is $\alpha=90^\circ$.
 $\int_{90^\circ}^{\alpha} \tau \, d\alpha = \int_{90^\circ}^{\alpha} pE \sin \alpha \, d\alpha$
n the vector form $U =$ **A Dipole in an Electric Field**
 Vork W must be done to change the
 u internation of the dipole in an external
 EXECUTE A PE dipole in a external
 EXECUTE A PE since angle for zero
 U = $W = \int_{90^\circ}^a \tau \, d\alpha = \int_{9$ **A Dipole in an Electric Field**

K W must be done to change the

Intation of the dipole in an external

I. This work is stored as **potential**

I. This work is stored as **potential**

<u>rgy</u> U. The reference angle for zero

$$
U = W = \int_{90^\circ}^{\alpha} \tau \, d\alpha = \int_{90^\circ}^{\alpha} pE \sin \alpha \, d\alpha = \left[-pE \cos \alpha \right]_{90^\circ}^{\alpha} = -pE \cos \alpha
$$

Rewritten in the vector form

$$
U=-\vec{p}\cdot\vec{E}
$$

Gauss's Law and Electric Flux

Let the surface be divided into elementary surfaces d*S*, small enough to be considered a plane. Electric field can be then taken as a constant for the surface. Electric flux through this area is $\begin{array}{ccc} \text{first} & \text{first} \\ \text{first} & \text{first} \end{array}$

$$
d\Phi_E = EdS\cos\alpha
$$

Where a is the angle between the vector \vec{E} and a unit vector \vec{n}_0 perpendicular to the surface d*S*.

The elementary surface can be also written as a vector

The formula for the electric flux can be then written in vector form

The total electric flux through an area *S* is given by

$$
d\vec{S} = \vec{n}_0 dS
$$

$$
d\Phi_E = \vec{E} \cdot d\vec{S}
$$

$$
\Phi_E = \iint \vec{E} \cdot d\vec{S}
$$

Gauss's Law and Electric Flux

Let us discuss a closed surface surrounding some volume. The electric flux through the surface is

$$
\Phi_E = \oint_S \vec{E} \cdot d\vec{S} = \oint_S E \cdot dS \cdot \cos \alpha
$$

The flux through a surface is proportional to the number of electric field lines through it. If the number of lines entering the volume is equal to the number of lines leaving the volume then there is no net flux out of this surface. The flux Φ_F will be nonzero only if some lines start or end within the volume. Since the lines start or end only on electric charge, the flux will be nonzero only if the *S* encloses a net charge. S Law and Electric F

d surface surrounding

urface is
 $\oint \vec{E} \cdot d\vec{S} = \oint_S E \cdot dS \cdot \cos \vec{S}$

e is proportional to the number of lines entering the

ng the volume then there

will be nonzero only if so

the lines start or **Example 10** Solution and Electric Flux

closed surface surrounding some volume. The
 $\Phi_E = \oint \vec{F} \cdot d\vec{S} = \oint \vec{F} \cdot dS \cdot \cos \alpha$

surface is proportional to the number of electric field

the number of lines entering the volu **EXECUTE SET AS A THE SET AS A THE SET AS A THEORY OF SET AS A THEORY OF SET AS A THOMAGE SET AS A THOUGHBOROOM BY CONTINUIST A THE SET AND INTERENT SURFACE (THE SET AND SERVING SERVING SERVING SERVING SERVING SERVING SER** and Electric Flux

ace surrounding some volume. The
 $\vec{s} = \oint_s \vec{E} \cdot d\vec{S} \cdot \cos \alpha$

bortional to the number of electric field

lines entering the volume is equal to

volume then there is no net flux out of

start or end o

The Gaus's law can be formulated by: *Electric flux through a closed surface equals to the net charge enclosed by that surface divided by a permittivity of free space*. *n*

$$
\oint_{S} \vec{E} \cdot d\vec{S} = \frac{\sum_{i=1}^{n} Q_i}{\varepsilon_0}
$$

Gauss's Law - deduction

Let's suppose that the surface *S* encloses charges $Q_1, Q_2, ..., Q_n$. If we consider elementary area dS₀, which is a projection of d*S* into the direction perpendicular to \vec{E} , we can write: **Gauss's Law - deductio**
uppose that the surface S
s charges Q_1 , Q_2 , ..., Q_n . If we
r elementary area dS₀, which
jection of dS into the direction
licular to \vec{E} , we can write:
 $\vec{S} = \oint_S \sum_{i=1}^n \vec{E}_i \cdot d\vec{$ **i Gauss's Law - dedu**

et's suppose that the surface S

ncloses charges Q_1 , Q_2 , ..., Q_n . If we

sa projection of dS into the direction

erpendicular to \vec{E} , we can write:
 $\oint_S \vec{E} \cdot d\vec{S} = \oint_S \sum_{i=1}^n \vec{E$

 $\sum_{n=1}^{n}$

$$
\bigoplus_{S} \vec{E} \cdot d\vec{S} = \bigoplus_{S} \sum_{i=1}^{n} \vec{E}_i \cdot d\vec{S} = \bigoplus_{S} \sum_{i=1}^{n} E_i \cdot dS \cos \alpha = \bigoplus_{S} \sum_{i=1}^{n} E_i \cdot dS_0
$$

Taking into account $dS_{0} = r_{i}^2 d\omega$ we can write

Gauss's Law - deduction
\nLet's suppose that the surface S
\nencloses charges
$$
Q_1, Q_2, ..., Q_n
$$
. If we
\nconsider elementary area dS_0 , which
\nis a projection of dS into the direction
\nperpendicular to \vec{E} , we can write:
\n
$$
\oiint_S \vec{E} \cdot d\vec{S} = \oiint_S \sum_{i=1}^n \vec{E}_i \cdot d\vec{S} = \oiint_S \sum_{i=1}^n E_i \cdot dS \cos \alpha = \oiint_S \sum_{i=1}^n E_i \cdot dS_0
$$
\nTaking into account $dS_0 = r_i^2 d\omega$ we can write
\n
$$
\oiint_S \sum_{i=1}^n E_i \cdot dS_0 = \int_0^{4\pi} \sum_{i=1}^n \frac{Q_i}{4\pi \varepsilon_0 r_i^2} r_i^2 d\omega = \sum_{i=1}^n Q_i \frac{1}{4\pi \varepsilon_0} \int_0^{4\pi} d\omega = \frac{\sum_{i=1}^n Q_i}{\varepsilon_0}
$$
\n
$$
\underbrace{\left(\oint_S \vec{E} \cdot d\vec{S} = \frac{\sum_{i=1}^n Q_i}{\varepsilon_0}\right)}_{1 \text{ steradian}}
$$

$$
\oint_{S} \vec{E} \cdot d\vec{S} = \frac{\sum_{i=1}^{n} Q_i}{\varepsilon_0}
$$

Application of Gauss's law – a point charge

We will examine the electric field around a positive point charge of magnitude *Q*. Considering a Gaussian surface in the form of a sphere at radius *r*, the electric field has the same magnitude at every point of the sphere and is directed outward.

Since the directions of \vec{E} and d \vec{S} are parallel also at every point of the sphere, we can simplify the Gaus's law.

r

Application of Gauss's law – a point charge
\nWe will examine the electric field around a positive point charge of magnitude Q.
\nConsidering a Gaussian surface in the form of a sphere at radius r, the electric field has the same magnitude at every point of the sphere and is directed outward.
\nSince the directions of
$$
\vec{E}
$$
 and $d\vec{S}$ are parallel also at every point of the sphere, we can simplify the Gauss's law.
\n
$$
\Phi_E = \oint_s \vec{E} \cdot d\vec{S} = \oint_s \vec{E} \cdot dS \cos \alpha = \oint_s \vec{E} \cdot dS = E \oint_s dS = E \cdot 4\pi r^2
$$
\n
$$
E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}
$$
\nThe electric field at radius r
$$
\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}
$$

Application of Gauss's law – a sphere of uniform charge

Now we have a dielectric sphere of radius *R* charged with uniform charge density *ρ* of the total magnitude *Q*.

When inside the sphere $(r < R)$ we are not surrounding the entire charge *Q* but only a part of it *Q*'.

Application of Gauss's law – a sphere of uniform charge
\nNow we have a dielectric sphere of radius R
\ncharged with uniform charge density
$$
\rho
$$
 of the
\ntotal magnitude Q.
\nWhen inside the sphere $(r < R$) we are not
\nsurrounding the entire charge Q but only a
\npart of it Q'.
\n
$$
Q' = \rho \frac{4}{3} \pi r^3
$$
 while
$$
Q = \rho \frac{4}{3} \pi R^3
$$

\n
$$
Q' = Q \frac{r^3}{R^3}
$$
 while
$$
Q = \rho \frac{4}{3} \pi R^3
$$

\n
$$
Q' = Q \frac{r^3}{R^3}
$$

$$
\Phi_E = \oiint_S \vec{E} \cdot d\vec{S} = E \cdot 4 \pi r^2 = \frac{Q}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \frac{r^3}{R^3}
$$

\nFinally we can write for $r < R$
$$
E = \frac{Q}{4 \pi \varepsilon_0 R^3} r
$$

Finally we can write for $r < R$

For $r > R$ we can use the same formula like for the point charge.

If Gauss's law – a sphere of uniform charge
\nelectric sphere of radius R
\nrm charge density
$$
\rho
$$
 of the
\nsphere (r\nthire charge Q but only a
\nwhile $Q = \rho \frac{4}{3} \pi R^3$
\n
$$
\Phi_E = \oint_S \vec{E} \cdot d\vec{S} = E \cdot 4 \pi r^2 = \frac{Q}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \frac{r^3}{R^3}
$$
\ne for r\n
$$
E = \frac{Q}{4 \pi \varepsilon_0 R^3} r
$$
\nuse the
\ne for the
$$
E = \frac{Q}{4 \pi \varepsilon_0 r^2}
$$

Q

r

 $d\vec{S}$

R

 \vec{E}

Application of Gauss's law – a charged wire

We are examinig electric field around a long wire charged with linear density τ . In this case we choose Gaussian surface as a cyliner centered around the wire or radius *r* and length *L*.

The electric flux through the cylinder caps is zero because there is right angle between \vec{E} and $d\vec{S}$. The flux passes only through the cylinder wall:

$$
\Phi_E = \oint_S \vec{E} \cdot d\vec{S} = E \cdot 2\pi r L = \frac{Q}{\varepsilon_0} = \frac{\tau \cdot L}{\varepsilon_0}
$$

The charge surrounded by the cylinder is *Q*=*τ·L*

The electric field at the distance *r* is given by

$$
E = \frac{\tau}{2\pi\varepsilon_0 r}
$$

Application of Gauss's law – an infinite charged plane

Now we have a charged plane of surface charge density σ . The Gaussian surface is a cylinder perpendicular to the plane. The electric flux through the cylinder wall is zero this because of the right angle between \vec{E} and d \vec{S} . The flux passes only through both cylinder caps of area *S*. *S E dS ES ES ES* Application of Gauss's law – an infinite charged plane
ow we have a charged plane of surface
arge density σ . The Gaussian surface
a cylinder perpendicular to the plane.
the electric flux through the cylinder wall
zero

$$
\Phi_E = \oint_S \vec{E} \cdot d\vec{S} = ES + ES = 2ES = \frac{\sigma S}{\varepsilon_0}
$$

The electric field is given by

We can note that the electric field around an infinite charged plane is constant and does not depend on the distance from the plane. The electric field is homogeneous.

 $2\varepsilon_0$ | \Box

 \mathcal{E}_{α} | \blacksquare

 σ | σ

 $E=\frac{0}{2}$

 $=\frac{0}{2}$

Application of Gauss's law – two parallel charged planes

This is the case of two parallel charged infinite planes of the same charge density σ and different charge signs.

We can see that the force lines to the left from positive plane and to the right from negative plane compensate each other so the electric field outside the planes must be zero.

We can also see that the density of force lines between planes is doubled compared to the single plane. This means that if the electric field caused by a single plane is *E*^s then the electric field *E* between planes must be 2*E*_s.

$$
E_s = \frac{\sigma}{2\varepsilon_0} \qquad \qquad \boxed{E = \frac{\sigma}{\varepsilon_0}}
$$

The electric field between infinite parallel planes is also constant and does not depend on the position between planes.

Electric Field and Conductors

1. **The net electric charge of a conductor resides entirely on its surface**. The mutual repulsion of like charges from Coulomb's Law demands that the charges be as far apart as possible, hence on the surface of the conductor.

2. **The electric field inside the conductor is zero**. Any net electric field in the conductor would cause charge to move since it is abundant and mobile. This would violate the condition of equilibrium: net force $= 0$.

3. **The external electric field at the surface of the conductor is perpendicular to that surface**. If there were a field component parallel to the surface, it would cause mobile charge to move along the surface, in violation of the assumption of equilibrium.

Electric Field Around a Conductor

Our Gaussian surface will be a small cylinder perpendicular to the surface. One cap of the cylinder will be just above the surface and the other just below the surface. **Electric Field Arour**

Our Gaussian surface will be a small

cylinder perpendicular to the surface.

One cap of the cylinder will be just

above the surface and the other just

below the surface.

The electric field is z **Electric Field Around a Cond**

Dur Gaussian surface will be a small

cylinder perpendicular to the surface.

One cap of the cylinder will be just

above the surface and the other just

below the surface.

The electric fi

The electric field is zero inside the conductor and is perpendicular to the surface just outside it.

The electric flux passes ony through the cylinder cap outside. If we choose the cylinder cap area *S* small enough (d*S*), the electric field

$$
\vec{E} \cdot d\vec{S} = E \cdot dS = \frac{dQ}{\varepsilon_0} = \frac{\sigma dS}{\varepsilon_0}
$$

where σ is the surface charge density at the place of cylinder.

The electric field at surface of conductor is then

$$
E = \frac{\sigma}{\varepsilon_0}
$$

Work and Potential in an Electric Field

If we place a unit charge Q₀ into an electric field \vec{E} around charge +*Q*, the force ${\vec F}_{\rm e}$ acting on it will be **Work and Potenti**
place a unit charge Q_0
c field \vec{E} around charge -
 \vec{F}_e acting on it will be
 \vec{F}_e =
vork done by this force to m
narge Q_0 from point K to L is
 $\int_K \vec{F}_e \cdot d\vec{r} = Q_0 \int_K \vec{E} \cdot d\vec{r}$
e

$$
\vec{F}_e = Q_0 \vec{E}
$$

The work done by this force to move the unit charge Q_{0} from point K to L is:

$$
W = \int\limits_K^L \vec{F}_e \cdot d\vec{r} = Q_0 \int\limits_K^L \vec{E} \cdot d\vec{r}
$$

An electric field due to a point charge *Q* is:

Work and Potential in an Electric Field
\nIf we place a unit charge
$$
Q_0
$$
 into an electric field \vec{E} around charge $+Q$, the force \vec{F}_e acting on it will be
\nforce \vec{F}_e acting on it will be
\n
$$
\vec{F}_e = Q_0 \vec{E}
$$
\nThe work done by this force to move the
\nunit charge Q_0 from point K to L is:
\n
$$
W = \int_{K}^{L} \vec{F}_e \cdot d\vec{r} = Q_0 \int_{K}^{L} \vec{E} \cdot d\vec{r}
$$
\nAn electric field due to a point charge Q is:
\n
$$
\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \vec{r}_0
$$
\nwhere \vec{r}_0 is a unit vector
\npointing outward from the Q.
\n
$$
W = Q_0 \int_{K}^{L} \frac{Q}{4\pi\varepsilon_0 r^2} \vec{r}_0 \cdot d\vec{r} = \frac{Q_0 Q}{4\pi\varepsilon_0} \int_{K}^{L} \frac{dr}{r^2} = \frac{-Q_0 Q}{4\pi\varepsilon_0} \left[\frac{1}{r} \right]_{r_k}^{r_L} = \frac{-Q_0 Q}{4\pi\varepsilon_0} \left[\frac{1}{r_L} - \frac{1}{r_K} \right]
$$

Work and Potential in an Electric Field

The work done by the force ${\vec F}_{\rm e}$ to move a the unit charge Q_{0} from point K to L is:

$$
W = \frac{-Q_0 Q}{4\pi \varepsilon_0} \left[\frac{1}{r_L} - \frac{1}{r_K} \right]
$$

We can note that the work *W* does not depend on the path taken, it depends only on the initial and final position. This also means that it we return back from L to K by a different path, the total work done will **Example 31**
 Example 31
 Example 31
 Example 31
 EXECT ARTLE AR $\frac{1}{r_L} - \frac{1}{r_K}$
path taken, it
means that it
vork done will
 $\vec{k} \cdot d\vec{r} = 0$ **ic Field**
 $\frac{-Q_0Q}{4\pi\varepsilon_0}\left[\frac{1}{r_L} - \frac{1}{r_K}\right]$

on the path taken, it

is also means that it

e total work done will
 $\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt$ **Exercise Field**
 $W = \frac{-Q_0 Q}{4\pi\varepsilon_0} \left[\frac{1}{r_L} - \frac{1}{r_K} \right]$

pend on the path taken, it

in. This also means that it

th, the total work done will

e field
 $\oint \vec{E} \cdot d\vec{r} = 0$ **ric Field**
 $\frac{-Q_0Q}{4\pi\varepsilon_0}\left[\frac{1}{r_L} - \frac{1}{r_K}\right]$

on the path taken, it

is also means that it

e total work done will
 $\oint \vec{E} \cdot d\vec{r} = 0$

d as work done by the **ctric Field**
 $=\frac{-Q_0Q}{4\pi\varepsilon_0}\left[\frac{1}{r_L}-\frac{1}{r_K}\right]$

ad on the path taken, it

This also means that it

the total work done will

eld
 $\oint \vec{E}\cdot d\vec{r}=0$

and as work done by the *P P P* **a extract Example 12**
 \therefore **extract by** $W = \frac{-Q_0 Q}{4 \pi \varepsilon_0} \left[\frac{1}{r_L} - \frac{1}{r_L} \right]$ **

not depend on the path**
 position. This also mead
 read to the total work
 ervative field. $\oint \vec{E} \cdot d\vec{r}$
 rge is defin a a Blectric Field
 B Example 1 Example 11 For Allen In Electric Field
 Example 12 For the path taken, it
 V does not depend on the path taken, it
 V does not depend on the path taken, it
 V does not depend on the path taken, it
 V a differe Electric Field
 $W = \frac{-Q_0 Q}{4\pi \varepsilon_0} \left[\frac{1}{r_L} - \frac{1}{r_K} \right]$

append on the path taken, it

on. This also means that it

ath, the total work done will

ve field.
 $\oint \vec{E} \cdot d\vec{r} = 0$

defined as work done by the

rom th **ential in an Electric Field**
 e to move a
 c to L is: $W = \frac{-Q_0 Q}{4 \pi \epsilon_0} \left[\frac{1}{r_L} - \frac{1}{r_K} \right]$
 V does not depend on the path taken, it

a different path, the total work done will

of conservative field.
 $\oint \vec{$

The potential energy *U* of the unit charge is defined as work done by the external force ${\vec F}_{\sf ext}$ in moving the charge ${\sf Q}_{\sf 0}$ from the reference point B to the point P. The force ${\vec F}_{\rm ext}$ must overcome the force ${\vec F}_{\rm e}$, so ${\vec F}_{\rm ext}$ = - ${\vec F}_{\rm e}$

The potential energy for $U_{\rm B}=0$ is then

$$
U = \int_{B}^{P} \vec{F}_{ext} \cdot d\vec{r} = -\int_{B}^{P} \vec{F}_{e} \cdot d\vec{r} = -Q_{0} \int_{B}^{P} \vec{E} \cdot d\vec{r}
$$

The **electric potential** is defined as potential energy per unit positive charge.

$$
\varphi = \frac{U}{Q_0} = -\int_{B}^{P} \vec{E} \cdot d\vec{r} \quad [V]
$$

Work and Potential in an Electric Field

The unit of electric potential is Volt [V]. If we want to express the potential difference between points K and L, we can write:

We can evaluate an expression for the We can evaluate an expression for the $\varphi_L = \varphi_K - \int\limits_{\Gamma}^L \vec{E} \cdot d\vec{r}$

For convenience we often place the reference point K to the infinity and we consider the potential to be zero here. Then we can simplify: **Work and Potential in an Electric Field**

The unit of electric potential is Volt [V]. If

we want to express the potential difference $\Delta \varphi = -\int_{K}^{L} \vec{E} \cdot d\vec{r} = \varphi_{L} - \varphi_{K}$

between points K and L, we can write:

W

To calculate a potential due to a group of charges we can use the superposition principle by simple adding particular

In case of uniform charge distribution we must integrate to determine the potential.

$$
\begin{aligned}\n\text{2ctric Field} \\
\Delta \varphi &= -\int_{K}^{L} \vec{E} \cdot d\vec{r} = \varphi_{L} - \varphi_{K} \\
\varphi_{L} &= \varphi_{K} - \int_{K}^{L} \vec{E} \cdot d\vec{r} \\
\varphi_{L} &= -\int_{\infty}^{L} \vec{E} \cdot d\vec{r} \\
\varphi &= \sum_{i=1}^{n} \varphi_{i} = \frac{1}{4\pi\varepsilon_{0}} \sum_{i=1}^{n} \frac{Q_{i}}{r_{i}} \\
\varphi &= \frac{1}{4\pi\varepsilon_{0}} \int \frac{dQ}{r}\n\end{aligned}
$$

$$
\varphi_L = \varphi_K - \int\limits_K^L \vec{E} \cdot d\vec{r}
$$

$$
\varphi_L=-\int\limits_{-\infty}^{L}\vec{E}\cdot d\vec{r}
$$

$$
\varphi = \sum_{i=1}^n \varphi_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^n \frac{Q_i}{r_i}
$$

$$
\varphi = \frac{1}{4\pi\varepsilon_0} \int \frac{dQ}{r}
$$

Electric Field and Potential

To find the relation between the electric field and potential we will consider two nearby points (*x,y,z*) and (*x+dx,y+dy,z+dz*). The potential change from the first point to the second is:

$$
d\varphi = \frac{\partial \varphi}{\partial x} dx + \frac{\partial \varphi}{\partial y} dy + \frac{\partial \varphi}{\partial z} dz
$$

We also know that

$$
p = -\vec{E} \cdot d\vec{r}
$$
 and
$$
d\vec{r} = \vec{i}dx + \vec{j}dy + \vec{k}dz
$$

Expressions for d*φ* can be compared

Electric Field and Potential
between the electric field and potential we will
by points
$$
(x,y,z)
$$
 and $(x+dx,y+dy,z+dz)$. The
m the first point to the second is:

$$
d\varphi = \frac{\partial \varphi}{\partial x} dx + \frac{\partial \varphi}{\partial y} dy + \frac{\partial \varphi}{\partial z} dz
$$

$$
d\varphi = -\vec{E} \cdot d\vec{r} \quad \text{and} \quad d\vec{r} = \vec{i}dx + \vec{j}dy + \vec{k}dz
$$

$$
\varphi \quad \frac{\partial \varphi}{\partial x} dx + \frac{\partial \varphi}{\partial y} dy + \frac{\partial \varphi}{\partial z} dz = -E_x dx - E_y dy - E_z dz
$$

$$
E_x = \frac{\partial \varphi}{\partial x}; \quad E_y = \frac{\partial \varphi}{\partial y}; \quad E_z = \frac{\partial \varphi}{\partial z}
$$

$$
\vec{E} = -\text{grad }\varphi
$$

We can finally write

$$
\vec{E} = -\text{grad }\varphi
$$

Example - electric potential around a point charge

Determine the potential *φ* at a distance r_L from a positive single point charge.

The magnitude of electric field due to a positive point charge is:

2 $4\pi\varepsilon_{\text{0}}^{\text{}}$ *Q E* $\pi \varepsilon_0 r$ $=$

The potential is given by

$$
\varphi_L = -\int_{-\infty}^{L} \vec{E} \cdot d\vec{r}
$$

The scalar product

Since the vector $d\vec{r}$ points in the negative direction of the *r* axis, we have to change the sign again, so

The potential is equal to

2 1 *L L L r r r L*

$$
\vec{E} \cdot d\vec{r} = E \cdot dr \cdot \cos 180^\circ = -E \cdot dr
$$

$$
\vec{E} \cdot d\vec{r} = E \cdot dr
$$

Example - electric potential around a point charge

The electric potential due to a point charge was deduced as

This is a dependence dropping hyperbolically and we can see from the formula that the surfaces with constant φ are concentric spherical areas.

These areas are called equipotential surfaces in the 3D case or equipotential lines in the 2D case. The equipotentials for the point charge configuration are shown on the picture by dashed lines.

Higher concentration of equipotentials mean higher electric field intensity and vice versa.

Graphical Interpretation of Electric Potential

In case of two large parallel charged plates of different signs we can see that equipotential surfaces are represented by planes parallel with the plates (dashed lines). The electric potential changes linearly here. **Interpretation of B**
ge parallel charged
gins we can see that
s are represented by
the plates (dashed
potential changes
 $\frac{r}{\epsilon_0}dr = \frac{-\sigma}{\epsilon_0} [r]_{x_A}^{x_B}$
 $-d;$ $\frac{\Delta \varphi = Ed}{\Delta \varphi}$
ws electric field and
ed) for the elect **Graphical Interpretat**
se of two large parallel
of different signs we can
stential surfaces are represe
is parallel with the plates
The electric potential
of here.
 $\int_{A}^{B} \vec{E} \cdot d\vec{r} = -\int_{x_A}^{x_B} \frac{\sigma}{\varepsilon_0} dr = \frac{-\sigma}{\v$ **Graphical Interpretatio**

of two large parallel ch

of different signs we can see

ential surfaces are represent

parallel with the plates (da

The electric potential cha

here.
 $\vec{E} \cdot d\vec{r} = -\int_{x_A}^{x_B} \frac{\sigma}{\varepsilon_0} dr = \$ *x A x* **Braphical Interpre**

of two large parall

different signs we catial surfaces are repr

rallel with the plate

ne electric potential

re.
 $\cdot d\vec{r} = -\int_{x_A}^{x_B} \frac{\sigma}{\varepsilon_0} dr = \frac{-c}{\varepsilon_0}$
 $\frac{\Delta}{\varepsilon_0}$

picture shows el **Graphical Interpretation of Electric**

of two large parallel charged

f different signs we can see that

parallel with the plates (dashed

The electric potential changes

here.
 $\vec{E} \cdot d\vec{r} = -\int_{x_A}^{x_B} \frac{\sigma}{\varepsilon_0} dr = \frac$ **Graphical Interpretation of Ele**
of two large parallel charged
different signs we can see that
ntial surfaces are represented by
arallel with the plates (dashed
he electric potential changes
ere.
 $\vec{z} \cdot d\vec{r} = -\int_{x_A}^{x$

$$
\varphi = -\int_{A}^{B} \vec{E} \cdot d\vec{r} = -\int_{x_{A}}^{x_{B}} \frac{\sigma}{\varepsilon_{0}} dr = -\frac{\sigma}{\varepsilon_{0}} \left[r \right]_{x_{A}}^{x_{B}}
$$
\n
$$
\varphi = \frac{\sigma}{\varepsilon_{0}} (x_{A} - x_{B}) = \frac{\sigma}{\varepsilon_{0}} d; \quad \Delta \varphi = Ed
$$

The next picture shows electric field and equipotentials (dashed) for the electric dipole.

There is always right angle between force lines and equipotential lines.

Capacitance

We have two large parallel conductive plates of the same charge density σ but different charge sign. The area of each plate is *S*, their distance is *d* and the space between them has permittivity *ε⁰ .*

0 $E = \frac{0}{10}$ 0⁺, area \mathcal{E}_0 We already know that the electric field between such plates is

The potential difference V= ϕ_{A} - ϕ_{B} is called voltage. The voltage between the plates is

The proportionality between voltage and charge can be expressed by a constant *C*

The constant *C* is called **capacitance** and for the parallel plate confirugation it is equal

d

Capacitance

A system of two isolated conductors is called a **capacitor** and the unit of its capacity is Farad. 1 $1F = \frac{1C}{4L}$ *C* **ELECTRIC** $F = \frac{16}{15}$ $=\frac{10}{11}$

$$
1F = \frac{1C}{1V}
$$

A single isolated conductor can also have a capacitance. It is defined as a ratio between charge Q and absolute potential ϕ . C= $\frac{Q}{\phi}$ φ .

The potential is relative to the zero potential in the infinity.

The unit of Farad is too large in practice. The capacity of the most common capacitors for electronics (on the picture) ranges from pF (10⁻¹² F) to μ F (10⁻⁶ F). An electric power line has a capacity to ground in units of nanofarads per kilometer $(1nF= 10^{-9} F)$.

Capacitor as Energy Storage

The energy stored in a capacitor is equal to work done to charge it. Charging means removing a charge from one plate and adding it to another. The work needed to transport a small amount of charge *dq* when a potential difference *V* is present on the plates is to charge it.
 d adding it to

of charge *dq*
 $dW = V dq$
 $\frac{q^2}{2} \int_0^Q = \frac{1}{2} \frac{Q^2}{C}$ to charge it.

adding it to

of charge dq
 $dW = V dq$
 $\frac{2}{2} \int_{0}^{2} = \frac{1}{2} \frac{Q^2}{C}$ Example is equal to work done to charge it.

ge from one plate and adding it to

sport a small amount of charge dq

sent on the plates is $dW = V dq$
 $\int_0^2 V dq = \frac{1}{C} \int_0^2 q dq = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^0 = \frac{1}{2} \frac{Q^2}{C}$
 U **Storage**

to work done to charge it.

one plate and adding it to

small amount of charge dq

the plates is $dW = V dq$
 $\frac{1}{C} \int_0^Q q dq = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q = \frac{1}{2} \frac{Q^2}{C}$
 $\frac{Q^2}{C}$; $Q = CV$
 $\frac{Q}{V}$
 $\frac{Q}{V}$ to charge it.

d adding it to

of charge dq
 $dW = V dq$
 $\frac{q^2}{2} \int_0^Q = \frac{1}{2} \frac{Q^2}{C}$
 $\frac{1}{2} \frac{QV}{C}$ **as Energy Storage**

citor is equal to work done to charge it.

charge from one plate and adding it to

transport a small amount of charge dq

s present on the plates is $dW = V dq$
 $W = \int_{0}^{Q} V dq = \frac{1}{C} \int_{0}^{Q} q dq = \frac{1}{C} \left[$ The to charge it.

Ind adding it to

It of charge dq
 $dW = V dq$
 $\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2} \frac{Q^2}{C}$

V
 $V_E = \frac{1}{2} QV$ **Energy Storage**

in is equal to work done to charge it.

arge from one plate and adding it to

ansport a small amount of charge dq

resent on the plates is $dW = V dq$
 $= \int_{0}^{Q} V dq = \frac{1}{C} \int_{0}^{Q} q dq = \frac{1}{C} \left[\frac{q^2}{2} \right]_{0$ **as Energy Storage**

citor is equal to work done to charge it.

charge from one plate and adding it to

transport a small amount of charge dq

s present on the plates is $dW = V dq$
 $W = \int_0^Q V dq = \frac{1}{C} \int_0^Q q dq = \frac{1}{C} \left[\frac{q$ **Energy Storage**

in is equal to work done to charge it.

arge from one plate and adding it to

ansport a small amount of charge dq

resent on the plates is $dW = V dq$
 $= \int_{0}^{Q} V dq = \frac{1}{C} \int_{0}^{Q} q dq = \frac{1}{C} \left[\frac{q^2}{2} \right]_{0$ **Capacitor as Enery**
 Example 3 energy stored in a capacitor is equality

in a potential difference V is present

ce V= $\frac{Q}{C}$ then the work
 $W = \int_{0}^{Q} V d\mu$

e is
 $W = \int_{0}^{Q} V d\mu$
 $W = U_E$

ed.
 $U_E = \frac{1}{2}CV^2$ afte one to charge it.

and adding it to

int of charge *dq*
 $dW = V dq$
 $\frac{q^2}{2} \left[\frac{q^2}{2} \right]_0^0 = \frac{1}{2} \frac{Q^2}{C}$
 $dV = \frac{1}{2} QV$

Since
$$
V = \frac{Q}{C}
$$
 then the work
done is

$$
W = \int_0^Q V dq = \frac{1}{C} \int_0^Q q dq = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q = \frac{1}{2} \frac{Q^2}{C}
$$

The work done is equal to electrical potential energy stored.

$$
W = U_E = \frac{1}{2} \frac{Q^2}{C}; \quad Q = CV
$$

$$
U_E = \frac{1}{2}CV^2
$$
 after subst

$$
\frac{1}{2}CV^2
$$
 after substituting $C = \frac{Q}{V}$ $U_E = \frac{1}{2}QV$

Energy Stored in a Parallel Plate Capacitor

From the previous we know relations for the electric field between charged parallel plates and for capacitance.

$$
E = \frac{V}{d} \implies V = Ed; \qquad C = \frac{\varepsilon_0 S}{d}
$$

y Stored in a Parallel Plate Capacitor

us we know relations for the electric field between

blates and for capacitance.
 $\frac{V}{d} \Rightarrow V = Ed;$ $C = \frac{\varepsilon_0 S}{d}$

ite for the $U = \frac{1}{2}CV^2 = \frac{1}{2} \frac{\varepsilon_0 S}{d} E^2 d^2 = \frac{1}{2} \varepsilon$ Then we can write for the potential energy 2 2 2 2 0 $d \frac{L}{a} = 2 \frac{c_0 L}{c_0 L} \frac{S}{a}$ \mathcal{E}_{0} \mathcal{S}_{-2} \mathcal{S}_{-1} \mathcal{S}_{-2} $C = \frac{\varepsilon_0 S}{d}$
 $c^2 = \frac{1}{2} \frac{\varepsilon_0 S}{d} E^2 d^2 = \frac{1}{2} \varepsilon_0 E^2 (S \cdot d)$

tween the plates. If we divide

, we obtain energy density w.
 $\left[\frac{J}{m^3} \right]$ $C = \frac{\varepsilon_0 S}{d}$
 $c^2 = \frac{1}{2} \frac{\varepsilon_0 S}{d} E^2 d^2 = \frac{1}{2} \varepsilon_0 E^2 (S \cdot d)$

tween the plates. If we divide

, we obtain energy density w.
 $\left[\frac{J}{m^3} \right]$

d for the case of the parallel

The product *S∙d* represents a volume between the plates. If we divide both sides of the equation by the volume, we obtain energy density *w.*

$$
\frac{U}{S \cdot d} = \frac{1}{2} \varepsilon_0 E^2 \qquad \qquad \boxed{w = \frac{1}{2} \varepsilon_0 E^2} \left[\frac{J}{m^3} \right]
$$

rgy Stored in a Parallel Plate Capacitor

vious we know relations for the electric field between
 V = $\frac{V}{d} \Rightarrow V = Ed$; $C = \frac{\varepsilon_0 S}{d}$

write for the $U = \frac{1}{2}CV^2 = \frac{1}{2} \frac{\varepsilon_0 S}{d} E^2 d^2 = \frac{1}{2} \varepsilon_0 E^2 (S \cdot d)$
 d **Stored in a Parallel Plate Capacitor**

we know relations for the electric field betwee

tes and for capacitance.
 $\frac{V}{d} \Rightarrow V = Ed;$ $C = \frac{\varepsilon_0 S}{d}$

for the $U = \frac{1}{2}CV^2 = \frac{1}{2} \frac{\varepsilon_0 S}{d} E^2 d^2 = \frac{1}{2} \varepsilon_0 E^2 (S \cdot d)$
 arallel Plate Capacitor
tions for the electric field between
acitance.
 d ; $C = \frac{\varepsilon_0 S}{d}$
 $\frac{1}{2}CV^2 = \frac{1}{2} \frac{\varepsilon_0 S}{d} E^2 d^2 = \frac{1}{2} \varepsilon_0 E^2 (S \cdot d)$
me between the plates. If we divide
plume, we obtain energy densit **Example 1 Plate Capacitor**

relations for the electric field between
 $\overline{C} = Ed;$ $C = \frac{\varepsilon_0 S}{d}$
 $U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 S}{d}E^2d^2 = \frac{1}{2}\varepsilon_0 E^2(S \cdot d)$

volume between the plates. If we divide

the volume, we ob **Parallel Plate Capacitor**

elations for the electric field between

spacitance.
 Ed ; $C = \frac{\varepsilon_0 S}{d}$
 $= \frac{1}{2}CV^2 = \frac{1}{2} \frac{\varepsilon_0 S}{d} E^2 d^2 = \frac{1}{2} \varepsilon_0 E^2 (S \cdot d)$

lume between the plates. If we divide

volume, we ob **Energy Stored**

om the previous we know

arged parallel plates and
 $E = \frac{V}{d} \Rightarrow$

on we can write for the

ential energy

be product S·d represents

th sides of the equation b
 $\frac{U}{S \cdot d} = \frac{1}{2} \varepsilon_0 E^2$

pough the f *u* in a Parallel Plate Capacitor

ow relations for the electric field

for capacitance.
 $V = Ed$; $C = \frac{\varepsilon_0 S}{d}$
 $U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 S}{d}E^2d^2 = \frac{1}{2}\varepsilon_0$

is a volume between the plates. If we

by the volume Although the formula for *w* was deduced for the case of the parallel plate capacitor, it is valid for any region where the electric field is present.

Dielectrics

In this part we will discuss insulators or dielectrics – materials, which do not conduct electricity.

What happens when we place a dielectric material in an electric field?

The answer depends on the type of used material and its molecules, we distinguish between

- a) Polar molecules
- b) Nonpolar molecules

Polar molecules – some molecules have nonsymmetric arrangement of their atoms. They have different position of the effective center of positive charge and negative charge due to this arrangement. As a result of this polar molecules indicate a dipole moment \vec{p} .

When such material is exposed to an external electric field, the dipole moments tend to align with the field.

Dielectrics

A typical polar molecule is water molecule H_2O . Oxygen nucleus with 8 protons is much stronger in attraction of electrons then hydrogen atoms with 1 proton each. This fact means that the position of the center of negative charge is closer to the oxygen nucleus then the position of the center of positive charge.

The dipole moment \vec{p} is marked in the picture. If we apply an external electric field, the moments will tend to align with the field, but the alignment will be only partial and dependent on the field intensity and temperature.

Dielectrics

Nonpolar molecules – we can find them in some gases. The molecules are symmetrical, so positions of centers of charge are the same and there is no dipole moment. We can see it on an oxygen molecule.

If such molecule is exposed to an electric field, negative electrons are pulled one way and positive nuclei are pulled the opposite way. This results in slight net displacement of the charge, so the dipole moment is present as well.

Conclusion: both types of molecules can acquire a dipole moment when placed in an electric field. The dielectrics become polarized.

Dielectrics and a Capacitor

If we have a parallel plate capacitor with vacuum between electrodes, its capacity was deduced as

If the space between plates is filled with dielectric the formula changes to

where ε^r is a **dielectric constant** or **relative permittivity** .

If we place the same charge *Q* on each capacitor and mark corresponding voltages V_{d} (dielectric) and $\bm{\mathsf{V}}_{0}$ (vacuum), we can write **D**
ave a parallel
m between electric despace between
ric the formula
 ϵ_r is a diele
re permittivity
place the same
ark correspond
cuum), we can
 $\epsilon_d V_d = C_0 V_0;$ **Dielectrics and a Capac**

ectrodes, its capacity

ectrodes, its capacity

en plates is filled with

a changes to
 lectric constant or $C_d =$
 lectric constant or $C_d =$
 lectric constant or $V_d = C_d =$
 lectric constan Dielectrics and a C:

arallel plate capacitor with

en electrodes, its capacity

s

stween plates is filled with

rmula changes to
 dielectric constant or
 rivity

a same charge Q on each

sponding voltages V_d (di **Dielectrics and a Cap**
 example the capacity with

ectrodes, its capacity
 len plates is filled with
 a changes to
 lectric constant or $\begin{bmatrix}$
 lectric constant or $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\$ **Dielectrics and a Capacity**

ve a parallel plate capacitor with

between electrodes, its capacity

luced as

acce between plates is filled with

c the formula changes to

c_r is a **dielectric constant** or
 C
 permitt

$$
Q = C_d V_d = C_0 V_0;
$$
 From the upper formulae we can deduce

so
$$
\frac{C_d}{C_0} = \varepsilon_r = \frac{V_0}{V_d} \implies V_d = \frac{V_0}{\varepsilon_r}
$$

0

d

 $\mathcal{E}_r = \frac{C_d}{\sigma}$

 C_{d}

 $C₀$

 $r =$ ^{α}

Dielectrics and a Capacitor

Our deduced formula for V_d shows us that the voltage between plates is lower in case of dielectric compared to $V_d = \frac{v_0}{c}$ the situation with vacuum for the same charge applied. *V* \mathcal{E}_{r} $=\frac{v_0}{v_0}$

If we also realize that electric field is directly proportional to the voltage for the parallel plate arrangement V=E*∙*d, it is obvious that also electric field is weaker in the dielectric. Why?

If we apply an external field to the parallel plate capacitor with dielectric between plates, molecules in the dielectric polarize, which induces an additional surface charge on the dielectric. The positive surface charge equal in magnitude to the negative one. Electric field caused by the surface charge has opposite direction to the external field.

0

r

Dielectrics and a Capacitor

If we mark the external field E_0 , and the electric field set by the surface charge E' , then the resultant electric field \vec{E} is given by $\vec{E} = \vec{E}_0 + \vec{E}'$

Conclusion: if we place a dielectric in an electric field, induced surface charges weaken the original electric field in the dielectric.

Relative Permittivity of Various Materials

We have again a comparison between parallel plate capacitor without dielectric (a) and with dielectric (b). If we apply Gauss's law to the (a) with Gaussian surface defined by the dashed line, we obtain

$$
\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = \varepsilon_0 ES = q \implies E_0 = \frac{q}{\varepsilon_0 S}
$$

If we apply Gauss's law to the (b) we obtain

$$
\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = \varepsilon_0 ES = q - q' \implies E = \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0 S} \tag{1}
$$

Where *q* is free charge and *q*' is induced charge.

Dielectrics, Capacitor and Gauss's Law

Now we recall previously mentioned formulae

$$
V_d = \frac{V_0}{\varepsilon_r} \quad and \quad V = E \cdot d \quad \Rightarrow \quad \varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}; \quad E = \frac{E_0}{\varepsilon_r}
$$

Considering $0^{\mathbf{D}}$ $E_{0} = \frac{q}{q}$ we can write $\left| I \right|$ $\varepsilon_0 S$ \sim $\frac{1}{2}$ $=\frac{q}{q}$ we can write r ^{$\boldsymbol{\sigma}$ 0^{$\boldsymbol{\omega}$}} $E = \frac{q}{q}$

auss's Law
 $\frac{6}{d}$; $E = \frac{E_0}{\varepsilon_r}$
 $\frac{q}{\varepsilon_r \varepsilon_0 S}$
 $\frac{q'}{\varepsilon_0 S}$; $\frac{q}{\varepsilon_r} = q - q'$ Combining with [1] we obtain $\frac{q}{q} = \frac{q}{q} - \frac{q}{q}$; $\frac{q}{q} = q - q$ **citor and Gauss's Law**

red formulae
 $\varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$

an write $E = \frac{q}{\varepsilon_r \varepsilon_0 S}$
 $\frac{q}{\varepsilon_r \varepsilon_0 S} = \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0 S}$; $\frac{q}{\varepsilon_r} = q - q'$
 $q - q' = q - q \left(1 - \frac{1}{\varepsilon_r}\right) = \frac$ $\frac{1}{q}$ $\mathcal{L} = \mathcal{L} \mathcal{L}$

Dielectrics, Capacitor and Gauss's Law
\nNow we recall previously mentioned formulae
\n
$$
V_d = \frac{V_0}{\varepsilon_r}
$$
 and $V = E \cdot d \implies \varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$
\nConsidering $E_0 = \frac{q}{\varepsilon_0 S}$ we can write $E = \frac{q}{\varepsilon_r \varepsilon_0 S}$
\nCombining with [1] we obtain $\frac{q}{\varepsilon_r \varepsilon_0 S} = \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0 S}$; $\frac{q}{\varepsilon_r} = q - q'$
\n $q' = q \left(1 - \frac{1}{\varepsilon_r}\right)$ $\varepsilon_0 \oint \oint \vec{E} \cdot d\vec{S} = q - q' = q - q \left(1 - \frac{1}{\varepsilon_r}\right) = \frac{q}{\varepsilon_r}$
\nFinally $\frac{\varepsilon_0 \oint \oint \varepsilon_r \vec{E} \cdot d\vec{S} = q}{\frac{\partial \oint \oint \varepsilon_r \vec{E} \cdot d\vec{S} = q}{\frac{\partial \vec{E} \cdot d\vec{S}}{\frac{\partial \vec{S}}}{\frac{\partial \vec{S}}{\frac{\partial \vec{S}}}{\frac{\partial \vec{S}}{\frac{\partial \vec{S}}}{\frac{\partial \vec{S}}}{\frac{\partial \vec{S}}{\frac{\partial \vec{S}}}{\frac{\partial \vec$

Finally

$$
\varepsilon_0 \bigoplus \varepsilon_r \vec{E} \cdot d\vec{S} = q
$$

 $r = \frac{E_0}{\varepsilon_r}$
 $\frac{q}{\varepsilon_r} = q - q'$
 $\frac{r}{\varepsilon_r}$
 $\frac{r}{r} = \frac{q}{\varepsilon_r}$
 ric constant ε_r are free charge **trics, Capacitor and Gauss's Law**

busly mentioned formulae
 $E \cdot d \Rightarrow \varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$
 \Rightarrow we can write $E = \frac{q}{\varepsilon_r \varepsilon_0 S}$
 \Rightarrow we obtain
 $\frac{q}{\varepsilon_r \varepsilon_0 S} = \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0$ $E = \frac{E_0}{\varepsilon_r}$
 $\left(\frac{q}{\varepsilon_r} - q\right)$
 $\left(\frac{q}{\varepsilon_r}\right) = \frac{q}{\varepsilon_r}$

tric constant ε_r and

the free charge q. uss's Law
 $\begin{aligned}\n\frac{1}{\sigma} &\frac{1}{\sigma} E = \frac{E_0}{\sigma} \\
\frac{1}{\sigma} &\frac{1}{\sigma} \frac{1}{\sigma} \frac{1}{\sigma} = q - q' \\
\left(1 - \frac{1}{\sigma} \right) &\frac{q}{\sigma} \frac{1}{\sigma} \\
\text{inlectric constant } \varepsilon_r \text{ and} \\
\text{only the free charge } q.\n\end{aligned}$ Capacitor and Gauss's Law

entioned formulae
 $\Rightarrow \varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$

we can write $\left[E = \frac{q}{\varepsilon_r \varepsilon_0 S} \right]$

in $\frac{q}{\varepsilon_r \varepsilon_0 S} = \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0 S}$; $\frac{q}{\varepsilon_r} = q - q'$
 $\cdot d\vec{$ cs, Capacitor and Gauss's Law

sly mentioned formulae
 $E \cdot d \Rightarrow \varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$

we can write $E = \frac{q}{\varepsilon_r \varepsilon_0 S}$

obtain $\frac{q}{\varepsilon_r \varepsilon_0 S} = \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0 S}$; $\frac{q}{\varepsilon_r} = q - q$ **Dielectrics, Capacitor and Gauss's Law**
 bow we recall previously mentioned formulae
 $V_d = \frac{V_0}{\varepsilon_r}$ and $V = E \cdot d \Rightarrow \varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$

considering $E_0 = \frac{q}{\varepsilon_0 S}$ we can write $E = \frac{$ **and Gauss's Law**
 *W*_a = $\frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$
 e $\boxed{E = \frac{q}{\varepsilon_r \varepsilon_0 S}}$
 $= \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0 S}$; $\frac{q}{\varepsilon_r} = q - q'$
 $= q - q \left(1 - \frac{1}{\varepsilon_r}\right) = \frac{q}{\varepsilon_r}$

ains a dielectric constant ε_r and
 Dielectrics, Capacitor and Gauss's Law

e recall previously mentioned formulae
 $\frac{V_0}{\varepsilon_r}$ and $V = E \cdot d \Rightarrow \varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$

lering $E_0 = \frac{q}{\varepsilon_0 S}$ we can write $\frac{F}{E} = \frac{q}{\varepsilon_r \v$ **Dielectrics, Capacitor and Gauss's Law**

we recall previously mentioned formulae
 $=\frac{V_0}{\varepsilon_r}$ and $V = E \cdot d \Rightarrow \varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$

sidering $E_0 = \frac{q}{\varepsilon_0 S}$ we can write $E = \frac{q}{\varepsilon_r \varepsilon_$ itor and Gauss's Law

ed formulae
 $\varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$
 γ write $E = \frac{q}{\varepsilon_r \varepsilon_0 S}$
 $\frac{q}{\varepsilon_0 S} = \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0 S}$; $\frac{q}{\varepsilon_r} = q - q'$
 $q - q' = q - q \left(1 - \frac{1}{\varepsilon_r}\right) = \frac{q}{\varepsilon$ *q q* **acitor and Gauss's Law**

band formulae
 $\varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$
 ε an write $E = \frac{q}{\varepsilon_r \varepsilon_0 S}$
 $\frac{q}{\varepsilon_r \varepsilon_0 S} = \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0 S}$; $\frac{q}{\varepsilon_r} = q - q'$
 $= q - q' = q - q \left(1 - \frac{1}{\v$ and Gauss's Law

umulae
 $\frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$

te $E = \frac{q}{\varepsilon_r \varepsilon_0 S}$
 $= \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0 S}$; $\frac{q}{\varepsilon_r} = q - q'$
 $t' = q - q \left(1 - \frac{1}{\varepsilon_r}\right) = \frac{q}{\varepsilon_r}$

tains a dielectric constant ε_r **Dielectrics, Capacitor and Gauss's Law**
 r we recall previously mentioned formulae
 $=\frac{V_0}{\varepsilon_r}$ and $V = E \cdot d \Rightarrow \varepsilon_r = \frac{V_0}{V_d} = \frac{E_0}{E_d}$; $E = \frac{E_0}{\varepsilon_r}$

ansidering $E_0 = \frac{q}{\varepsilon_0 S}$ we can write $E = \frac{q}{\varepsilon$ We can note that the flux integral contains a dielectric constant *ε^r* and that the charge within Gaussian surface is only the free charge *q*. Induced charge is hidden in the *ε^r* .

Polarization and Electric Displacement

Previously we deduced an equation

After small arrangements

 $\overline{\mathcal{S}}$

tion and Electric Displacement

\niced

\n
$$
\frac{q}{\varepsilon_{r}\varepsilon_{0}S} = \frac{q}{\varepsilon_{0}S} - \frac{q'}{\varepsilon_{0}S}
$$
\nints

\n
$$
\frac{q}{\varepsilon_{0}S} = \frac{q}{\varepsilon_{r}\varepsilon_{0}S} + \frac{q'}{\varepsilon_{0}S}; \quad \boxed{\frac{q}{S} = \varepsilon_{0} \frac{q}{\varepsilon_{r}\varepsilon_{0}S} + \frac{q'}{S}}
$$
\ninduced charge per area.

\narization P.

\nWe can rewrite the

\n
$$
\frac{q}{\varepsilon_{0}S} \qquad \text{we can rewrite the} \quad \frac{q}{\varepsilon_{0}S} = \varepsilon_{0}E + P
$$
\nectric displacement D.

\n
$$
D = \frac{q}{\varepsilon_{0}S} \qquad \boxed{\frac{C}{\varepsilon_{0}S}}
$$

placement

\n
$$
\frac{1}{S}; \quad \frac{q}{S} = \varepsilon_0 \frac{q}{\varepsilon_r \varepsilon_0 S} + \frac{q'}{S}
$$
\na.

\n
$$
P = \frac{q'}{S} \left[\frac{C}{m^2} \right]
$$
\nthe

\n
$$
\frac{q}{S} = \varepsilon_0 E + P
$$
\n
$$
D = \frac{q}{S} \left[\frac{C}{m^2} \right]
$$
\nis

\n
$$
\frac{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}
$$

The last term $\frac{q\prime}{q}$ $\overline{\mathcal{S}}$ is the induced charge per area. The last term $\frac{1}{s}$ is the induced charge per area.
We call it **electric polarization P**. $P = \frac{q'}{S}$

q E $\mathcal{E}_{r} \mathcal{E}_{0} S$ Realizing that $E = \frac{q}{q}$ we can rewrite the equation to The term $\frac{q}{q}$ is called **electric displacement D**. **Polarization and Electric Displacement**

Previously we deduced $\frac{q}{\varepsilon_r \varepsilon_0 S} = \frac{q}{\varepsilon_0 S} - \frac{q'}{\varepsilon_0 S}$

After small arrangements $\frac{q}{\varepsilon_0 S} = \frac{q}{\varepsilon_r \varepsilon_0 S} + \frac{q'}{\varepsilon_0 S}$; $\frac{q}{S} = \varepsilon_0 \frac{q}{\varepsilon_r \varepsilon_0 S$

$$
\frac{q}{S} = \varepsilon_0 E + P
$$

$$
D = \frac{q}{S} \left[\frac{C}{m^2} \right]
$$

$$
\vec{\overline{D}} = \varepsilon_0 \vec{E} + \vec{P}
$$

Polarization and Electric Displacement

The displacement and polarization can be demonstrated on a parallel plate capacitor with combination of a gap and dielectric between electrodes. There are some important findings.

- 1. Vector \vec{D} is associated only with the free charge.
- 2. Vector \vec{P} is associated only with the polarization charge.
- 3. Vector \vec{E} is associated with all present charges.
- 4. Vector \vec{P} vanishes outside the dielectric, vector \vec{D} is not affected by the environment and vector \vec{E} has different magnitudes in the gap and in the dielectric.

Polarization and Electric Displacement

 $0^ \alpha$, μ

 $0^{\mathcal{O}}$ c_r

; $E=\frac{E_0}{\sqrt{2}}$

If we combine previously deduced formulae

we can write

In vector form it is

The polarization can be written as

q q E **Electric Displacement**
 $E_0 = \frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $\frac{q}{S} = \varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $\overrightarrow{D} = \varepsilon_0 \varepsilon_r \overrightarrow{E}$
 $P = \frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{$ ctric Displacement
 $\frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $\overline{\varepsilon_0 E_0} = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $\overline{\varepsilon_0 \varepsilon_r \vec{E}}$
 $\frac{r'}{s} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r} \right) =$
 $\frac{r$ **lectric Displacement**
 $=\frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $=\varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $=\frac{\varepsilon_0 \varepsilon_r \vec{E}}{S}$
 $=\frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r} \right) =$
 \frac ctric Displacement
 $\frac{q}{\epsilon_0 S}$; $E = \frac{E_0}{\epsilon_r}$ and $D = \frac{q}{S}$
 $\frac{d}{dE_0 E_0} = \epsilon_0 \epsilon_r E \implies D = \epsilon_0 \epsilon_r E$
 $\frac{d}{dE} = \frac{q}{S} \left(1 - \frac{1}{\epsilon_r} \right) = \epsilon_0 \epsilon_r E \left(1 - \frac{1}{\epsilon_r} \right) = E - \epsilon_0 E = \epsilon_0 (\epsilon_r - 1) E$ **E**₀ = $\frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $\frac{q}{S} = \varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $\frac{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}}{\vec{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r}\right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r}\right) =$
 $\frac{\varepsilon_0 \varepsilon_r E - \v$ $\frac{q}{s} = \varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \Rightarrow D = \varepsilon_0 \varepsilon_r E$ Electric Displacement
 $=\frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $=\varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $=\frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r}\right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r}\right) =$
 $=\frac{q'}{S_0 \varepsilon_r E - \varepsilon_0 E} = \varepsilon_0 (\$ *D* Electric Displacement
 $E_0 = \frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $\frac{q}{S} = \varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $\frac{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}}{S}$
 $P = \frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left($ Electric Displacement
 $=\frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $=\varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $=\frac{\varepsilon_0 \varepsilon_r \vec{E}}{S}$
 $=\frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r}\right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r}\right) =$ $0^{\mathcal{L}}$ ^r $\left| \begin{array}{c} 1 \\ 1 \end{array} \right|$ Example 1
 $\begin{aligned}\n&= \frac{q}{\varepsilon_0 S}; \quad E = \frac{E_0}{\varepsilon_r} \quad and \quad D \\
&= \varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \quad \Rightarrow \quad D = \varepsilon_0 \n\end{aligned}$
 $\begin{aligned}\n&= \frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r} \right) \\
&= \varepsilon_0 \varepsilon_r E - \varepsilon_0 E = \varepsilon_0 (\varepsilon_r -$ **Isomeriary**
 $\frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $E \Rightarrow D = \varepsilon_0 \varepsilon_r E$
 $\frac{1}{\varepsilon_r}$ = $\varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r}\right) =$
 $\varepsilon_0 (\varepsilon_r - 1) E$
 $q \Rightarrow \left| \frac{\oint \vec{D} \cdot d\vec{S} = q}{\oint \vec{D} \cdot d\vec{S}} = \frac{q}{\varepsilon_r} \right|$ cement
 $\frac{0}{r}$ and $D = \frac{q}{S}$
 $\Rightarrow D = \varepsilon_0 \varepsilon_r E$
 $\left(1 - \frac{1}{\varepsilon_r}\right) = (\varepsilon_r - 1)E$
 $\Rightarrow \left[\oint \vec{D} \cdot d\vec{S} = q\right]$ **acement**
 $\frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $\vec{\varepsilon} \Rightarrow D = \varepsilon_0 \varepsilon_r E$
 $\left(1 - \frac{1}{\varepsilon_r}\right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r}\right) = \varepsilon_0 (\varepsilon_r - 1) E$
 $\Rightarrow \left| \oint \vec{D} \cdot d\vec{S} = q \right|$ *r r₆ =* $\frac{q}{\varepsilon_0 S}$ *;* $E = \frac{E_0}{\varepsilon_r}$ *and* $D = \frac{q}{S}$ *
* $\frac{q}{S} = \varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$ *
* $\frac{\overline{D} = \varepsilon_0 \varepsilon_r \overline{E}}{P = \frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r} \right) =$ *
* ectric Displacement
 $\frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and D
 $\varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0$
 $\frac{\varepsilon_0 \varepsilon_r \vec{E}}{S}$
 $\frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r} \right)$
 $\therefore E - \varepsilon_0 E = \varepsilon_0 (\varepsilon$ ctric Displacement
 $\frac{q}{\epsilon_0 S}$; $E = \frac{E_0}{\epsilon_r}$ and $D = \frac{q}{S}$
 $\delta E_0 = \epsilon_0 \epsilon_r E \implies D = \epsilon_0 \epsilon_r E$
 $\frac{q}{\epsilon_r \overline{E}}$
 $\frac{q}{\epsilon_r \epsilon_r} = \frac{q}{\epsilon_s} \left(1 - \frac{1}{\epsilon_r}\right) = \epsilon_0 \epsilon_r E \left(1 - \frac{1}{\epsilon_r}\right) = E - \epsilon_0 E = \epsilon_0 (\epsilon_r - 1) E$
 $\frac{(\epsilon_r - 1)\overline{E}}$ ment

and $D = \frac{q}{S}$
 $> D = \varepsilon_0 \varepsilon_r E$
 $\varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r}\right) =$
 $-1)E$
 $\oint \vec{D} \cdot d\vec{S} = q$ Electric Displacement
 $\sum_{\substack{\overline{c}_0,\overline{S}}} \frac{q}{\overline{c}_0}$; $E = \frac{E_0}{\overline{\varepsilon}_r}$ and $D = \frac{q}{S}$
 $= \varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $= \frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r} \right) =$
 $= \v$ d Electric Displacement
 $E_0 = \frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $\frac{q}{S} = \varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $\overline{D} = \varepsilon_0 \varepsilon_r \overline{E}$
 $P = \frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{$ **d Electric Displacement**
 $E_0 = \frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $\frac{q}{S} = \varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $\frac{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}}{S}$
 $P = \frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left($ $\vec{P} = \varepsilon_0 (\varepsilon_r - 1) \vec{E}$ Electric Displacement
 $=\frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$
 $=\varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$
 $=\frac{q'}{S} = \frac{q}{S} \left(1 - \frac{1}{\varepsilon_r} \right) = \varepsilon_0 \varepsilon_r E \left(1 - \frac{1}{\varepsilon_r} \right) =$
 $=\frac{\varepsilon_0 \varepsilon_r E - \varepsilon_0 E}{\varepsilon_$ **Polarization and Electric Displacement**

If we combine previously $E_0 = \frac{q}{\varepsilon_0 S}$; $E = \frac{E_0}{\varepsilon_r}$ and $D = \frac{q}{S}$

we can write
 $\frac{q}{S} = \varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E \implies D = \varepsilon_0 \varepsilon_r E$

In vector form it is
 $\frac{|\overline{D} =$

0 and Γ

r

And in vector form

Finally we can formulate the Gauss's law for the

$$
\varepsilon_0 \oint \varepsilon_r \vec{E} \cdot d\vec{S} = q \implies \boxed{\oint \vec{D} \cdot d\vec{S} = q}
$$

Summary – what we have learnt

Coulomb's law

A force acting on a charge in an electric field

Gauss's law

Relations between potential and electric field

Relations between electric field, electric displacement and polarization

Energy stored in a capacitor

Energy density of electric field

$$
\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r_{21}^2} \vec{r}_{21}^0
$$

$$
\vec{F} = Q\vec{E}
$$

have learnt
\n
$$
\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r_{21}^2} \vec{r}_{21}^0
$$
\n
$$
\vec{F} = Q\vec{E}
$$
\n
$$
\oiint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}; \quad \oiint_S \vec{D} \cdot d\vec{S} = Q
$$
\n
$$
\varphi = -\int_A^B \vec{E} \cdot d\vec{r}; \quad \vec{E} = -\operatorname{grad} \varphi
$$
\n
$$
\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}; \quad \vec{D} = \varepsilon_0 \vec{E} + \vec{P}
$$
\n
$$
U_E = \frac{1}{2} C V^2
$$
\n
$$
w = \frac{1}{2} \varepsilon_0 E^2
$$

$$
\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}; \quad \vec{D} = \varepsilon_0 \vec{E} + \vec{P}
$$

$$
U_E = \frac{1}{2} C V^2
$$

$$
w = \frac{1}{2} \varepsilon_0 E^2
$$

Example – Electric field on the axis of charged ring

Given: ring radius *R*, charge *q*. *E*(z)=?

Magnitude of the electric field at the point P due to the element *ds* from the Coulomb's law:

$$
dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\tau \cdot ds}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\tau \cdot ds}{z^2 + R^2}
$$

Horizontal component of $d\vec{E}$ is compensated by the element on the opposite side of the ring, so only

Example – Electric field on the axis of charged ring
\nGiven: ring radius R, charge q. E(z)=?
\nMagnitude of the electric field at the point P due to
\nthe element ds from the Coulomb's law:
\n
$$
dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\tau \cdot ds}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\tau \cdot ds}{z^2 + R^2}
$$
\nHorizontal component of $d\vec{E}$ is compensated by the
\nelement on the opposite side of the ring, so only
\nvertical component $dE\cos\Theta$ can be taken into account.
\n
$$
\cos\Theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}};
$$
\n
$$
dE\cos\Theta = \frac{1}{4\pi\varepsilon_0} \frac{z \cdot \tau}{(z^2 + R^2)^{3/2}} ds \qquad E = \int dE\cos\Theta
$$

Example – Electric field on the axis of charged ring

Example – Electric field on the axis of charged ring
\n
$$
E = \int dE \cos \Theta = \int \frac{1}{4\pi \varepsilon_0} \frac{z \cdot \tau}{(z^2 + R^2)^{3/2}} ds = \frac{1}{4\pi \varepsilon_0} \frac{z \cdot \tau}{(z^2 + R^2)^{3/2}} \int_0^{2\pi R} ds
$$
\n
$$
E = \frac{1}{4\pi \varepsilon_0} \frac{z \cdot \tau \cdot 2\pi R}{(z^2 + R^2)^{3/2}} \qquad \boxed{E = \frac{1}{4\pi \varepsilon_0} \frac{z \cdot q}{(z^2 + R^2)^{3/2}}}
$$
\nA graph for $q=4$ µC, $R=2$ cm. Important points and limits
\n
$$
E = 0 \quad \text{for} \quad z = 0
$$
\n
$$
E = 0 \quad \text{for} \quad z \to \infty
$$
\nThe maximum for $z = \frac{R}{2}$

$$
E = \frac{1}{4\pi\varepsilon_0} \frac{z \cdot \tau \cdot 2\pi R}{(z^2 + R^2)^{3/2}} \qquad \boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{z \cdot q}{(z^2 + R^2)^{3/2}}}
$$

A graph for *q*= 4 μC, *R*= 2 cm. Important points and limits

is field on the axis of charged ring

\n
$$
\frac{z \cdot \tau}{\frac{2}{3} + R^2 \cdot 3^{3/2}} \, ds = \frac{1}{4\pi \varepsilon_0} \frac{z \cdot \tau}{(z^2 + R^2 \cdot 3^{3/2}} \int_0^{2\pi R} \, ds
$$
\n
$$
E = \frac{1}{4\pi \varepsilon_0} \frac{z \cdot q}{(z^2 + R^2 \cdot 3^{3/2})}
$$
\ncm.

\nImportant points and limits

\n
$$
E = 0 \quad \text{for} \quad z = 0
$$
\n
$$
E = 0 \quad \text{for} \quad z \to \infty
$$
\nThe maximum for $z_m = \frac{R}{\sqrt{2}}$

\n
$$
\text{For } z >> R
$$
\nor $z >> R$

\nor $z = \frac{1}{4\pi \varepsilon_0} \frac{q}{z^2}$

For *z* >>*R*

$$
E = \frac{1}{4\pi\varepsilon_0} \frac{q}{z^2}
$$

Example – Electric field on the axis of charged disc

Given: disc radius *R*, surface charge density σ. *E*(z)=?

A charge contribution from the elementary ring of radius *r* is

Using a formula for charged ring we can write

$$
dE = \frac{1}{4\pi\varepsilon_0} \frac{z \cdot \sigma \cdot 2\pi r \, dr}{(z^2 + r^2)^{3/2}} = \frac{z \cdot \sigma}{4\varepsilon_0} \frac{2r \, dr}{(z^2 + r^2)^{3/2}}
$$

$$
E = \int dE = \frac{z \cdot \sigma}{4\varepsilon_0} \int_0^R (z^2 + r^2)^{-3/2} 2r \, dr
$$
 We substit $x = z^2 + r^2$

We substitute

$$
x = z^2 + r^2; \quad dx = 2r dr
$$

Example – Electric field on the axis of charged disc
\nven: disc radius *R*, surface charge density
$$
\sigma
$$
. $E(z)=?$
\ncharge contribution from the elementary ring of
\ndius *r* is\n
$$
dq = \sigma dS = \sigma 2\pi r dr
$$
\n
$$
dE = \frac{1}{4\pi \varepsilon_0} \frac{z \cdot \sigma \cdot 2\pi r dr}{(z^2 + r^2)^{3/2}} = \frac{z \cdot \sigma}{4\varepsilon_0} \frac{2r dr}{(z^2 + r^2)^{3/2}}
$$
\n
$$
E = \int dE = \frac{z \cdot \sigma}{4\varepsilon_0} \int_0^R (z^2 + r^2)^{-3/2} 2r dr
$$
\nWe substitute\n
$$
x = z^2 + r^2; \quad dx = 2r dr
$$
\n
$$
E = \frac{z \cdot \sigma}{4\varepsilon_0} \int_0^R x^{-3/2} dx = \frac{z \cdot \sigma}{4\varepsilon_0} \left[\frac{x^{-1/2}}{-\frac{1}{2}} \right] = \frac{z \cdot \sigma}{4\varepsilon_0} \left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R
$$

Example – Electric field on the axis of charged disc

Example – Electric field on the axis of charged disc
\n
$$
E = \frac{-z \cdot \sigma}{2\epsilon_0} \Big[(z^2 + r^2)^{-1/2} \Big]_0^R = \frac{-z \cdot \sigma}{2\epsilon_0} \Big[(z^2 + R^2)^{-1/2} - \frac{1}{z} \Big] = \frac{\sigma}{2\epsilon_0} \Big[1 - \frac{z}{\sqrt{z^2 + R^2}} \Big]
$$
\n
$$
\frac{E = \frac{\sigma}{2\epsilon_0} \Big[1 - \frac{z}{\sqrt{z^2 + R^2}} \Big] \Big[1 - \frac{z}{\sqrt{z^2 + R^2}} \Big] \Big]
$$
\n
$$
\text{Important points and limits}
$$
\nA graph for $\sigma = 5 \text{ }\mu\text{C/cm}^2$, $R = 10 \text{ cm}$.
\n
$$
E = \frac{\sigma}{2\epsilon_0} \qquad \text{for} \qquad z = 0
$$
\n
$$
E = \frac{\sigma}{2\epsilon_0} \qquad \text{for} \qquad R \to \infty
$$
\n
$$
E = 0 \qquad \text{for} \qquad z \to \infty
$$
\n
$$
E = 0 \qquad \text{for} \qquad z \to \infty
$$
\n
$$
E = 0 \qquad \text{for} \qquad z \to \infty
$$

A graph for $\sigma = 5 \mu C/cm^2$, $R = 10 \text{ cm}$.

Important points and limits

$$
E = \frac{\sigma}{2\varepsilon_0} \quad \text{for} \quad z = 0
$$
\n
$$
E = \frac{\sigma}{2\varepsilon_0} \quad \text{for} \quad R \to \infty
$$

$$
E = 0 \quad \text{for} \quad z \to \infty
$$

A conducting sphere of radius R_1 is surrounded by a concentric dielectric layer of outer radius R_2 and permitivity *ε*₂. The surrounding medium has permitivity *ε*1<*ε*² . Find the dependence of electric displacement, electric field and potential on the distance from the center of the sphere charged to the charge $Q = D(r)=?$, $E(r)=?$, $\varphi(r)=?$ **Example – Concentric spheres**

sphere of radius R_1 is surrounded

c dielectric layer of outer radius R_2
 ε_2 . The surrounding medium has
 ε_2 . Find the dependence of electric

electric field and potential o **Example – Co**

conducting sphere of radius R_1

a concentric dielectric layer of c

and permitivity ε_2 . The surroundin-

splacement, electric field and postance from the center of the sph

e charge Q. D(r)=?, E(r) **Example – Concentric spheres**

conducting sphere of radius R_1 is surrounded

y a concentric dielectric layer of outer radius R_2

and permittivity ε_2 . The surrounding medium has

dermittivity $\varepsilon_1 < \varepsilon_2$. Fi **Example – Concentric spheres**
phere of radius R_1 is surrounded
c dielectric layer of outer radius R_2
 ε_2 . The surrounding medium has
 ε_2 . Find the dependence of electric
electric field and potential on the
 Example – Co
conducting sphere of radius R_1
y a concentric dielectric layer of c
nd permitivity ε_2 . The surroundin
ermitivity $\varepsilon_1 < \varepsilon_2$. Find the depende
isplacement, electric field and po
istance from the **Example – Concentric spheres**

Example – Concentric spheres

a concentric dielectric layer of outer radius R_2

permitivity ε_2 . The surrounding medium has

mitivity $\varepsilon_1 \le \varepsilon_2$. Find the dependence of electric **Example – Concentric spheres**

sphere of radius R_1 is surrounded

ric dielectric layer of outer radius R_2

ty ε_2 . The surrounding medium has
 $\varepsilon \varepsilon_2$. Find the dependence of electric

the center of the sph Example – Concentric spheres

va conclucting sphere of radius R_1 is surrounded

va a concentric dielectric layer of outer radius R_2

and permitivity $\varepsilon_1 < \varepsilon_2$. The surrounding medium has

issince from the cente

Electric displacement

Gauss's law simplified for the concentric arrangement

$$
\oiint_{S} \vec{D} \cdot d\vec{S} = Q; \quad D \oiint_{S} dS = D \cdot 4\pi r^2 = Q
$$

Since the charge inside the sphere is zero, then

Electric field

$$
\oint_{S} \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon}; \quad E \oiint_{S} dS = E \cdot 4\pi r^{2} = \frac{Q}{\varepsilon}
$$

Electric field in the sphere

$$
E = \frac{Q}{4\pi r^2 \varepsilon}
$$

 $4\pi r^2\varepsilon\big|\;$ sphere is zero Q Charge inside the $E = \frac{Q}{1 + \frac{2}{\pi}}$ charge inside sphere is zero, so

$$
E = 0
$$
 for r $\lt R_1$

Electric field in the dielectric

$$
\frac{Q}{2\pi r^2 \varepsilon} \begin{array}{c} \text{Change inside the} \\ \text{sphere is zero, so} \end{array}
$$
\n
$$
\frac{E = 0}{2\pi r^2 \varepsilon} \begin{array}{c} \text{force in the sphere is zero, so} \\ \text{force in the sphere is zero, so} \end{array}
$$
\n
$$
\frac{E = \frac{Q}{4\pi r^2 \varepsilon_2}}{\sqrt{Q}}
$$

ntric spheres
\n**is** zero, so
$$
\boxed{E=0} \text{ for r < R_1}
$$
\n
$$
\boxed{Q}
$$
\n
$$
\boxed{4\pi r^2 \varepsilon_2}
$$
\n
$$
\boxed{\text{for } R_1 < r < R_2}
$$
\n
$$
\boxed{\text{for } r > R_2}
$$

Electric field outside the dielectric

$$
E = \frac{Q}{4\pi r^2 \varepsilon_1} \qquad \text{for } r > R_2
$$

$$
\left|\frac{Q}{r^2c}\right| \qquad \text{for } r > R_2
$$

Example – Concentric spheres
\nElectric field
\nin the sphere
$$
E = \frac{Q}{4\pi r^2 \epsilon}
$$
 Charge inside the sphere is zero, so $\boxed{E=0}$ for r=R₁
\nElectric field in the dielectric
\nElectric field outside the dielectric
\n $E = \frac{Q}{4\pi r^2 \epsilon_2}$ for R₁rR₂
\n $E = \frac{Q}{4\pi r^2 \epsilon_1}$ for r>R₁R₂
\n $F = \frac{Q}{4\pi r^2 \epsilon_1}$ for r>R₂
\n $F = \frac{Q}{4\pi r^2 \epsilon_1}$ for r>R₂
\n $\phi(r) = 0 + \frac{Q}{4\pi \epsilon_2} \int_{R_1}^{R_2} \frac{dr}{r^2} + \frac{Q}{4\pi \epsilon_1} \int_{R_2}^{R_1} \frac{dr}{r^2} = \frac{Q}{4\pi \epsilon_2} \left[-\frac{1}{r} \right]_{R_1}^{R_2} + \frac{Q}{4\pi \epsilon_1} \left[-\frac{1}{r} \right]_{R_2}^{R_3}$
\n $\phi(r) = \frac{Q}{4\pi \epsilon_2} \left[\frac{1}{R_1} - \frac{1}{R_2} \right] + \frac{Q}{4\pi \epsilon_1} \left[\frac{1}{R_2} \right]$ $\frac{Q}{Q} = \text{const.}$

Potential

$$
\boxed{\text{for } R_1 < r < R_2 \qquad \varphi(r) = \int\limits_r^{R_2} E \cdot dr + \int\limits_{R_2}^{\infty} E \cdot dr = \frac{Q}{4\pi \varepsilon_2} \int\limits_r^{R_2} \frac{dr}{r^2} + \frac{Q}{4\pi \varepsilon_1} \int\limits_{R_2}^{\infty} \frac{dr}{r^2}}
$$

Example – Concentric spheres
\nPotential
\n
$$
\frac{\text{for } R_1 < r < R_2}{\text{for } R_1 < r < R_2} \qquad \varphi(r) = \int_r^{R_2} E \cdot dr + \int_{R_2}^{\infty} E \cdot dr = \frac{Q}{4\pi \varepsilon_2} \int_r^{\frac{R_2}{2}} \frac{dr}{r^2} + \frac{Q}{4\pi \varepsilon_1} \int_{R_2}^{\infty} \frac{dr}{r^2}
$$
\n
$$
\varphi(r) = \frac{Q}{4\pi \varepsilon_2} \left[-\frac{1}{r} \right]_{r}^{R_2} + \frac{Q}{4\pi \varepsilon_1} \left[-\frac{1}{r} \right]_{R_2}^{\infty} = \frac{Q}{4\pi \varepsilon_2} \left[\frac{1}{r} - \frac{1}{R_2} \right] + \frac{Q}{4\pi \varepsilon_1} \left[\frac{1}{R_2} \right]
$$
\n
$$
\frac{\varphi(r) = \frac{Q}{4\pi \varepsilon_1 r} + const}{\varphi(r) = \int_r^{\infty} E \cdot dr = \frac{Q}{4\pi \varepsilon_1} \int_r^{\infty} \frac{dr}{r^2} = \frac{Q}{4\pi \varepsilon_1} \left[-\frac{1}{r} \right]_{r}^{\infty} = \frac{Q}{4\pi \varepsilon_1 r}
$$
\n
$$
\frac{\varphi(r) = \frac{Q}{4\pi \varepsilon_1 r}}{\frac{Q}{4\pi \varepsilon_1 r}}
$$

$$
\text{for } r > R_2
$$
\n
$$
\varphi(r) = \int_r^\infty E \cdot dr = \frac{Q}{4\pi \varepsilon_1} \int_r^\infty \frac{dr}{r^2} = \frac{Q}{4\pi \varepsilon_1} \left[\frac{-1}{r} \right]_r^\infty = \frac{Q}{4\pi \varepsilon_1 r}
$$

$$
\varphi(r) = \frac{Q}{4\pi\varepsilon_1 r}
$$

Example – Coaxial capacitor

Determine a formula for the capacitance of a cylindrical capacitor of radii R_1 , R_2 , length *I* and permitivity of the dielectric *ε*. We assume that inner conductor is charged to *+Q* and outer one to *–Q*. **Example**

betermine a formula

radii R_1 , R_2 , length l and p

the dielectric ε . We assumer conductor is charged to

tue one to $-Q$.

om the Gauss's law
 $\oint_C \vec{E} \cdot d\vec{S} = E \oint_C dS = E \cdot 2\pi$

ne voltage between the **Example – Coaxial capacito**

Fraction continuous continuous continuous continuous continuous dil R_1 , R_2 , length *l* and permitivity

re dielectric ε . We assume that

r conductor is charged to +Q and

r one to -**Example** -

nine a formula for

tance of a cylindrical cap

dielectric ε . We assum

conductor is charged to +

one to -Q.

the Gauss's law
 $\cdot d\vec{S} = E \bigoplus_{s} dS = E \cdot 2\pi r l$

oltage between the cylinde
 $\int_{1}^{2} E \cdot dr = \$ **Example** -

nine a formula for

itance of a cylindrical ca

i R_1 , R_2 , length *l* and periodielectric ε . We assum

conductor is charged to +

one to -Q.

the Gauss's law
 $\cdot d\vec{S} = E \oint_C dS = E \cdot 2\pi r l$

oltage betw **Example – Coa**
mine a formula for the
itance of a cylindrical capacito
dil R_1 , R_2 , length *l* and permitivity
e dielectric ε . We assume tha
conductor is charged to +Q and
one to –Q.
the Gauss's law
 $\vec{r} \cdot d\vec{$ **Example – Coa**
mine a formula for the
itance of a cylindrical capacito
dil R_1 , R_2 , length l and permitivity
e dielectric ε . We assume tha
conductor is charged to +Q and
one to –Q.
the Gauss's law
 $\vec{r} \cdot d\vec{S}$ **Example – Coaxial capacitor**

Determine a formula for the

apacitance of a cylindrical capacitor

of radii R_1 , R_2 , length *I* and permitivity

of the dielectric ε . We assume that

more conductor is charged to + **Example – Coaxial capa**
formula for the
cylindrical capacitor
gth *l* and permitivity
for and
charged to +Q and
law
 $dS = E \cdot 2\pi r l = \frac{Q}{\varepsilon}$
en the cylinders
 $\frac{p}{\varepsilon l} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{Q}{2\pi \varepsilon l} [\ln r]_{R_1}^{R_2} =$

 $E=\frac{Q}{\sigma}$

 $=\frac{Q}{2}$

reil and *read to the method* α

Q

From the Gauss's law

$$
\oint_{S} \vec{E} \cdot d\vec{S} = E \oint_{S} dS = E \cdot 2\pi r l = \frac{Q}{\varepsilon} \qquad E = \frac{Q}{2\pi \varepsilon r l}
$$

The voltage between the cylinders

$$
V = \int_{R_1}^{R_2} E \cdot dr = \frac{Q}{2\pi \varepsilon l} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{Q}{2\pi \varepsilon l} \Big[\ln r \Big]_{R_1}^{R_2} = \frac{Q}{2\pi \varepsilon l} \ln \frac{R_2}{R_1}
$$

The capacitance

$$
C = \frac{Q}{V} = \frac{2\pi\epsilon l}{\ln \frac{R_2}{R_1}}
$$

mm, ϵ
such c

 $\ln \frac{n_2}{n}$ such cable is $\begin{array}{|l|} \hline 2\pi\varepsilon l \end{array}$ Example: I=1 m, R₁=1 mm, R₂=3 $\overline{V} = \frac{1}{\sqrt{R_2}}$ mm, $\epsilon_r = 3$. The capacitance of R_1 \sim \sim \sim \sim \sim \sim \sim such cable is C= 152 pF.

Example – Spherical capacitor

Determine a formula for the capacitance of a spherical formed by two concentric spheres of radii *R*₁ and *R*₂. P permitivity of the dielectric is *ε*. We assume that inner sphere is charged to *+Q* and outer one to *–Q*. **Example – Spherical cap**

mine a formula for the capacitance of

cal formed by two concentric spheres of ra

d R_2 . P permitivity of the dielectric is ε . V

ne that inner sphere is charged to +Q and

one to –Q.

pr **Example – Spherical cap**

mine a formula for the capacitance of

cal formed by two concentric spheres of ra

d R_2 . P permitivity of the dielectric is ε . V

ne that inner sphere is charged to +Q and

one to -Q.

pr **Example – Spherical capacito**

rmine a formula for the capacitance of a

rical formed by two concentric spheres of radii

rical formed by two concentric spheres of radii

reductions we know $E = \frac{Q}{4\pi\epsilon r^2}$

revious de **Example – Spherical capacito**
mine a formula for the capacitance of a
ical formed by two concentric spheres of radii
of R_2 . P permitivity of the dielectric is ε . We
me that inner sphere is charged to +Q and
one to **Example – Spherical capacitor**

Determine a formula for the capacitance of a

spherical formed by two concentric spheres of radii
 R_1 and R_2 . P permitivity of the dielectric is ε . We

assume that inner sphere i **Example – Spherical capacitor**

termine a formula for the capacitance of a

nerical formed by two concentric spheres of radii

and R_2 . P permittivity of the dielectric is ε . We

sume that inner sphere is charged t **Example – S**
mula for the coy two concentri
mitivity of the c
er sphere is changed the spheres
ductions we kno
een the spheres
 $\frac{2}{\pi \varepsilon} \int_{R_1}^{R_2} \frac{dr}{r^2} = \frac{Q}{4\pi \varepsilon} \left[\frac{E_2}{R_1} - R_1 \right]$ **Example – Sp**
rmula for the ca
by two concentric
errmitivity of the die
er sphere is chan
eductions we know
veen the spheres
 $Q \rvert_{\overline{R}\epsilon}^{R_2} \rvert_{\overline{R}\epsilon}^{2} = \frac{Q}{4\pi\varepsilon} \left[-\frac{Ex}{\pi\varepsilon} \right]^{R_1}$
 $\frac{Ex}{\pi\varepsilon}$
The
 $C =$ **Example – Spherical callated to the capacitance of the capacitance of** R_1 **and** R_2 **. P permitivity of the dielectric is** ε **, ssume that inner sphere is charged to +Q uter one to -Q.

from previous deductions we know Example – Spherical cap**
mine a formula for the capacitance of
ical formed by two concentric spheres of rad R_2 . P permitivity of the dielectric is ε . N
ne that inner sphere is charged to +Q a
one to -Q.
previous d **Example – Spherical capacitor**

Final formula for the capacitance of a

Final formed by two concentric spheres of radii

and R_2 . P permitivity of the dielectric is ε . We

are the timer sphere is charged to +0 and

From previous deductions we know

$$
E = \frac{Q}{4\pi\epsilon r^2} \qquad \qquad \boxed{\frac{2R_1}{2R_2}}
$$

The voltage between the spheres

Example – Spherical capacitor
\nDetermine a formula for the capacitance of a spherical formed by two concentric spheres of radii
$$
R_1
$$
 and R_2 . P permittivity of the dielectric is ε . We assume that inner sphere is charged to +Q and outer one to $-Q$.
\nFrom previous deductions we know
$$
E = \frac{Q}{4\pi\varepsilon r^2}
$$
\nThe voltage between the spheres
\n
$$
V = \int_{R_1}^{R_2} E \cdot dr = \frac{Q}{4\pi\varepsilon} \int_{R_1}^{R_2} \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon} \left[-\frac{1}{r} \right]_{R_1}^{R_2} = \frac{Q}{4\pi\varepsilon} \left[\frac{1}{R_1} - \frac{1}{R_2} \right] = \frac{Q}{4\pi\varepsilon} \frac{R_2 - R_1}{R_1 R_2}
$$
\nThe capacitance
\nThe capacitance
\n
$$
\frac{E \times \text{ample: } R_1 = 1 \text{ cm, } R_2 = 2 \text{ cm, } \varepsilon_r = 3. \text{ The capacitance of such capacitor is} \qquad C = 667 \text{ pF.}
$$

The capacitance

$$
C = \frac{Q}{V} = 4\pi\varepsilon \frac{R_1 R_2}{R_2 - R_1}
$$

Example: $R_1=1$ cm, $R_2=2$ cm, $\epsilon_r=3$. The capacitance of such capacitor is C= 667 pF.