MEASUREMENT OF THE ACCELERATION DUE TO THE GRAVITY WITH A REVERSIBLE PENDULUM AND STUDY OF THE GRAVITATIONAL FIELD.

OBJECT

- To measure acceleration due to gravity for Prague.
- To plot a graph of the dependence of τ_{0d} and $\tau_{0\,u}$ on the lens position.
- To evaluate combined uncertainty of the measurement

THEORY:

Reversion pendulum.

Reversion pendulum is a special type of the physical pendulum. *Physical pendulum* is a rigid body rotating (swinging) around fixed horizontal axis, which do not pass through the center of mass of the pendulum HS.

A torque acting on the pendulum can be evaluated as

$$M = -mgd\sin\varphi$$

where *m* is a mass of the pendulum, *d* is the distance between the axis of rotation and the center of mass and φ is angular displacement from the equilibrium. The minus sign means, that the torque acts in the opposite direction compared to the displacement, i.e. it pushes the pendulum back to the equilibrium.

Equation of motion for a body rotating around fixed axis is

$$J\varepsilon = J\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = M$$

where ε is angular acceleration and *J* is moment of inertia of the body about the chosen axis.

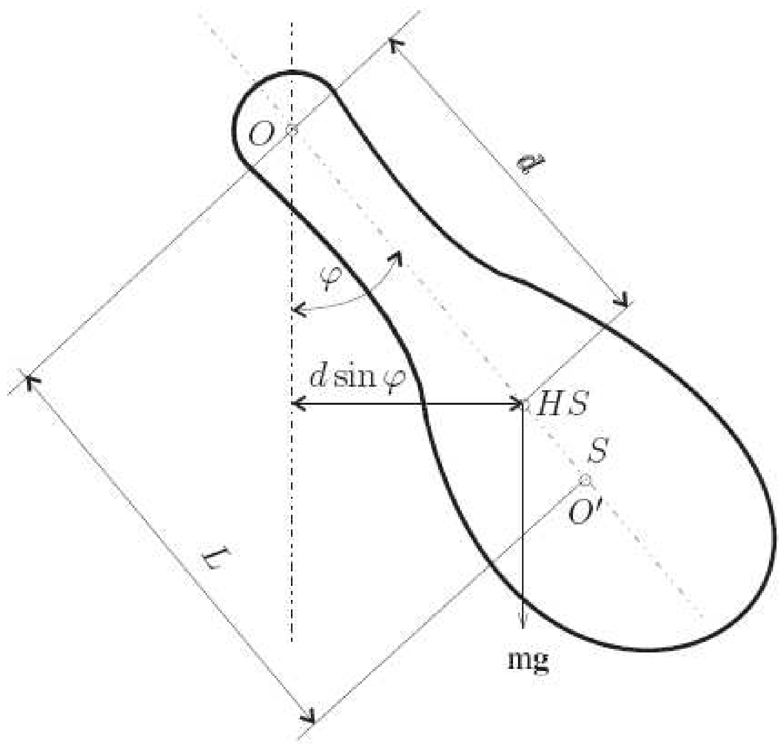

Fig. 1. Reversible pendulum

Substituting for the M from the first equation we obtain equation of motion for our pendulum:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} + \frac{mgd}{J}\sin\varphi = 0$$

Solution of such nonlinear differential equation leads to an elliptic integral, but we can simplify it by substituting sinφ ~ φ. Our error is less than about 0.05% for angular displacement smaller than 5°. We obtain simple linear differential equation:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} + \omega_0^2\varphi = 0$$

where $\omega_0^2 = mgd/J$ is angular frequency of the pendulum. A swing time (half of the period) is

$$\tau_0 = \frac{T_0}{2} = \pi\sqrt{\frac{J}{mgd}}$$

Taking into account initial conditions

$$\varphi(t=0) = \varphi_m \ , \quad \left.\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right|_{t=0} = 0$$

we obtain for the angular displacement

$$\varphi = \varphi_m \cos\omega t$$

Lets introduce the term *mathematical pendulum* now. It is a particle of mass *m* hanging on the massless fibre of the length *l*. The moment of inertia of such pendulum is $J = ml^2$ hence the swing time is now

$$\tau_0 = \pi\sqrt{\frac{ml^2}{mgl}} = \pi\sqrt{\frac{l}{g}}$$

Note that the swing time of mathematical pendulum does not depend on the mass *m*. Comparing swing time formulae for the physical and mathematical pendulums we can say that the length of the mathematical pendulum is equivalent to the *J/md* ratio for the physical one.

$$L = \frac{J}{md}$$

The quantity *L* is *reduced length* of physical pendulum. **Reduced length of the physical pendulum is equal to the length of mathematical pendulum, which has the same swing time**. For the swing time of physical pendulum we can write

$$\tau_0 = \pi\sqrt{\frac{L}{g}}$$

If J_0 is moment of inertia of the pendulum about an axis passing through its center of mass, then the moment of inertia about an axis parallel to the one passing through the center of mass is according to the parallel axis theorem

$$J = J_0 + md^2$$

Lets remind, that *m* is a mass of the pendulum and *d* is distance between center of mass and axis of rotation.
Dependence between the square of the swing time and the distance *d* can be written as

$$\tau_0^2 = \pi^2 (J_0 + md^2)/mgd$$

The dependence is depicted on the following figure

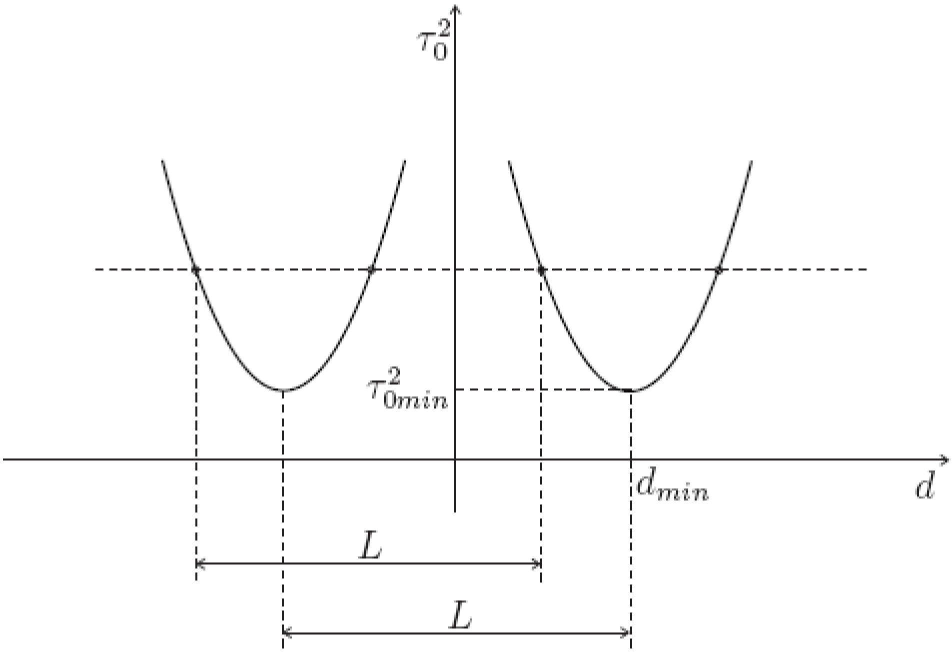

Fig. 2 Dependence of the square of the swing time on the distance *d*

The minimum of this function representing the fastest motion of the pendulum corresponds to

$$d_{min} = \sqrt{\frac{J_0}{m}}$$

If we put the d_{min} into the equation $J = J_0 + md^2$, then we obtain

$$d_{min} = \sqrt{\frac{J}{2m}}$$

Considering d_{min} in the formula $L = \frac{J}{md}$ we finally obtain $d_{min} = \frac{L}{2}$, i.e. the reduced length can be calculated as $L = 2d_{min}$

The figure 2 shows us, that there are four ways of the pendulum suspension with the same swing time for $\tau_0 > \tau_{0min}$, each two on either side from the center of mass. If we find nonsymmetrical position of two axes, where the swing time is the same, then the distance between those axes is equal to the reduced length *L*.
Construction of the reversible pendulum is based on this theory. It is created by a metal bar with two axes *O* and *O'* formed by three-sided prisms. Heavy lens is mounted on the bar and can move along it. We find such position of the lens that the swing times are identical for the pendulum hanging on either axis.

Then we can say that the distance *OO'* is equal to the reduced length L and after measuring swing time τ_0 we are able to calculate the acceleration due to gravity

$$g = \frac{\pi^2 L}{\tau_0^2}$$

PROCEDURE

1. Turn on the swing counter with stopwatch and set the second switch to the START position.
2. Set the lens position to the closest distance from the three-sided prism. Don't forget to gently tighten the securing matrix. Hang the pendulum with the bottom position of the lens. Displace the pendulum from the equilibrium to the stop and release it.
3. Press the button "NULOVANI" (RESET). The swing counter resets and after passing the next equilibrium it begins to measure time and to count swings. After every multiple of 100 swings the counter stops for a short period (about 5 secs).
4. Read the time $100\tau_{0d}$. Hang the pendulum with the upper position of the lens, displace it and let it swing. Read the time $100\tau_{0u}$.
5. Increase the distance between the lens and the three-sided prism by one rotation of the lens (representing 1 mm shift) and repeat steps 2,3 and 4. Plot both swing times into the graph as a function of the lens position .
6. Proceed with the measurement until the curves of τ_{0d} and τ_{0u} cross each other, which means that upper and bottom swing times are identical.
7. With the pendulum "tuned" this way measure the swing time for 500 swings for bottom and upper position of the lens.
8. Calculate an average value from $500\tau_{0d}$ and $500\tau_{0u}$ and calculate the acceleration due to gravity.
9. Calculate combined uncertainty of the measurement.
10. Compare your results with the accepted value of the acceleration due to gravity for Prague.