FREE FALL STUDY AND ACCELERATION DUE TO GRAVITY MEASUREMENT

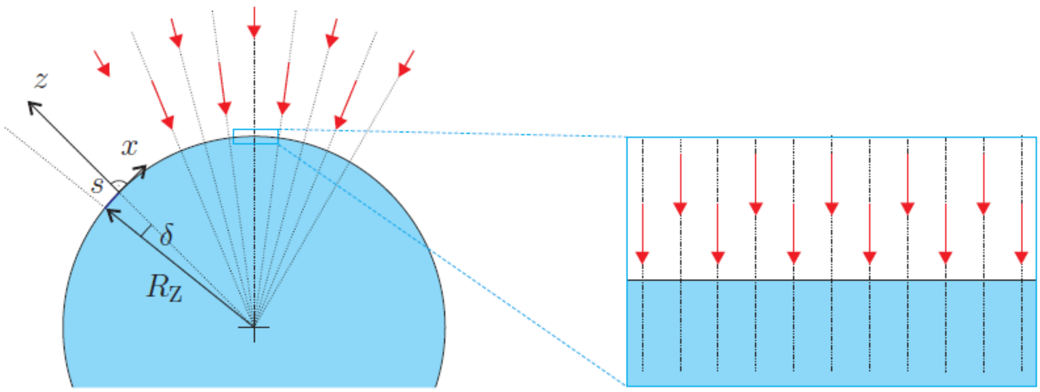
OBJECT

1. Measure the free fall time of two steel balls of various diameters.
2. Create a graph with the dependence of the free fall time on the height; calculate the acceleration due to gravity g and its uncertainty. Compare the result with the accepted value for Prague.

THEORY

When examining the free fall in the gravitational field of the Earth we firstly accept some simplifications.

a) *We consider the gravitational field of the Earth homogeneous.* We can afford this in all situations where the free fall height is much smaller than the radius of the Earth ($R_E = 6378$ km).



central force field homogeneous force field

Figure 1. Approximation of the central force field by the homogeneous one

b) *We disregard the Coriolis' force.* This force is given by the following formula

$$\vec{F}_C = 2m\vec{v} \times \vec{\omega} ,$$

where m is mass of a body moving on the surface of the Earth, $\boldsymbol{v}$ is its velocity (its direction is identical with the direction of the $\boldsymbol{g}$ in case of the free fall – see Figure 2) and $\boldsymbol{\omega}$ is angular velocity of the Earth's rotation. Direction of the Coriolis' force is to the East in case of a free fall on the northern hemisphere.

The acceleration associated with the Coriolis' force and its magnitude can be evaluated by

$$\vec{a}_C = 2\vec{v} \times \vec{\omega}; \quad a_C = 2v\,\omega \sin\varphi ,$$

where the φ is latitude of the free fall place.

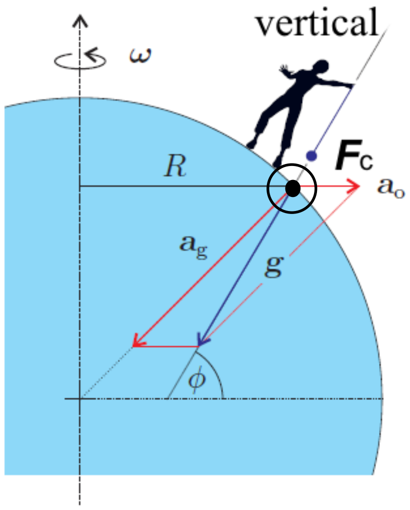


Figure 2. Coriolis' force and centrifugal acceleration

If we suppose the maximum free fall height for our experiment to be h_{max} = 50 cm, then the maximum velocity reached by the falling body is $v_{max} = \sqrt{2gh_{max}} = \sqrt{2 \cdot 9.81 \cdot 0.5} = 3.13\,\mathrm{m \cdot s^{-1}}$

The magnitude of the acceleration associated with the Coriolis' force for such velocity is

$$a_C = 2v_{max}\,\omega \sin\varphi = 2 \cdot 3.13 \cdot \frac{2\pi}{3600 \cdot 24} \sin 50° = 3.5 \times 10^{-4}\ \mathrm{m \cdot s^{-2}}$$

This value represents about 0,035 ‰ (per mile) of the g, so we can easily disregard it.

c) Centrifugal force of the Earths' rotation

An acceleration corresponding to this force must be subtracted from the acceleration due to gravity calculated from the Newton's law of universal gravitation to be able to obtain comparable results with our measurement. The formula for the subtraction is

$$g_{real} = a_g - a_0 = \frac{GM_E}{R_E^{\ 2}} - \frac{4\pi^2}{T^2} R \ ,$$

where a_g is acceleration due to gravity from the Newton's law, a_0 is centrifugal acceleration, $G = 6.67 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$ is gravitational constant, mass of the Earth $M_E = 6 \times 10^{24}$ kg, radius of the Earth R_E = 6378 km, R is the distance of the body from the axis of Earth‘s rotation (Figure 2), T is period of the Earth‘s rotation.

We can go around the necessity of this calculation by comparing our result with the accepted value of the g for Prague.

d) We disregard the resistance of the air

The resistance of the air is described by the empiric Newton's relation

$$F_r \approx \frac{1}{2} C \rho_0 S v^2 ,$$

where C is a shape coefficient (for a sphere we can consider $C \approx 0.5$), ρ_0 is density of the fluid (for the air $\rho_0 \approx 1.3\ \mathrm{kg \cdot m^{-3}}$), S is cross-sectional area of the moving body and v is its velocity.

If we take into account steel ball of the diameter D = 15 mm, its cross-sectional area is about $S = 177\ \mathrm{mm^2}$ and its mass is about m = 13.8 g. The resistive force for these values results in $F_r = 5.63 \times 10^{-4}$ N. The weight of the ball is $G = m \cdot g = 0.0138 \cdot 9.81 = 0.135$ N.

The resistive force for previously calculated v_{max} is about 0.4 % of the weight, so we can disregard it as well.

Motion in the gravitational field of the Earth.

Accepting the previous simplifications we can easily examine motion of a particle in the gravitational field of the Earth. The equation of motion is

$$m\frac{d^2\vec{r}}{dt^2} = m\frac{d\vec{v}}{dt} = m\vec{g} \quad \Rightarrow \quad \frac{d\vec{v}}{dt} = \vec{g} \qquad [1]$$

If we consider the $\boldsymbol{g}$ vector a constant, we can simply integrate equation [1]

$$d\vec{v} = \vec{g}dt \quad \Rightarrow \quad \int_{v_0}^{v} d\vec{v}' = \int_0^t \vec{g}dt' \quad \Rightarrow \quad \vec{v} = \frac{d\vec{r}}{dt} = \vec{g}t + \vec{v}_0, \qquad [2]$$

where the $\boldsymbol{v}_0$ is the particle velocity at the moment $t = 0$ (initial condition). From the equation [2] we can also integrate the position vector dependence on time.

$$d\vec{r} = (\vec{g}t + v_0)dt \quad \Rightarrow \quad \int_{r_0}^{r} d\vec{r}' = \int_0^t (\vec{g}t' + \vec{v}_0)dt' \quad \Rightarrow \quad \vec{r} = \frac{1}{2}\vec{g}t^2 + \vec{v}_0 t + \vec{r}_0, \qquad [3]$$

where $\boldsymbol{r}_0$ is the position vector at the moment $t = 0$ (initial condition)

Example: a vertical throw

We throw a body from the place described by a position vector $\boldsymbol{r}_0$= [0, 0, h] (from the height h) with initial velocity $\boldsymbol{v}_0$=[0, 0, v_0]. Since the acceleration due to gravity is given by $\boldsymbol{g}$=[0, 0, -g] we obtain by substituting for each component in the equation [3]

$$x(t) = 0, \qquad y(t) = 0 \qquad z(t) = -\frac{1}{2}gt^2 + v_0 t + h. \qquad [4]$$

The free fall time we can obtain by substituting $z = 0$ in the equation [4]

$$x(t) = 0, \qquad y(t) = 0 \qquad h = \frac{1}{2}g\tau^2 - v_0\tau \qquad [5]$$

The experiment

The experiment consists of releasing a steel ball from various heights h_i and measuring corresponding free fall times τ_i. We use a variant of the Atwood's machine for the measurement – Fig. 3. A steel ball (1) is held by a neodymium magnet in the upper part of the device. By pressing the trigger (2) we release the ball, which falls on an impact detector (3). The information about the release and about the impact is transferred to the timer via connectors (4) and (5). The holder of the ball can be repositioned up or down along a bar with centimeter scale (6). The screw (7) serves for releasing or tightening of the entire holder of the ball.

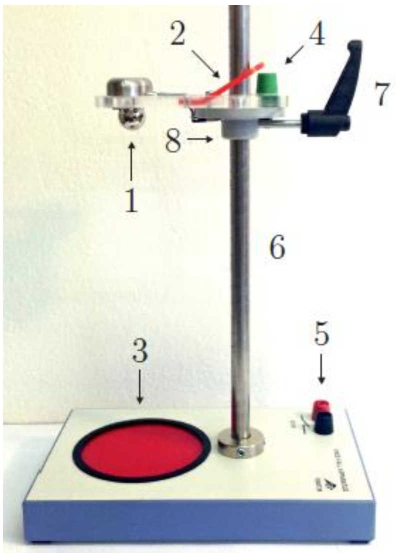


Figure 3. The free fall measuring device

Timer / counter is shown on the Fig. 4. We can select a mode of the device by the rotational switch (1). The fall time measurement can be performed at the position Δt_{AB}, the resolution of the measurement can be set to 0.1 s, 1 ms or 0.1 ms. By pressing the RESET button (2) we can reset the value on the display. The contact associated with the ball release mechanism must be connected to the IN START/COUNT clamp (3) and the impact sensor must be connected to the IN STOP clamp (4). Take care for clamp colors. The device can be turned on and off by a switch on the power cable.

Figure 4. Timer / counter

PROCEDURE

1. Choose one of steel balls and measure its diameter and mass.
2. Measure fall times τ_i for at least 10 various heights h_i.
3. Calculate the magnitude of the acceleration due to gravity for Prague including is uncertainty.
4. Create a graph with the dependence of the free fall time on the height (approximate measured data by the second order polynomial).
5. Repeat steps 1. – 4. for a ball of different diameter.

PROCESSING OF MEASURED DATA

According to the procedure we gained a set of data represented by h_i and τ_i pairs. Now we must approximate this set by a second order polynomial

$$h = a_2\tau^2 + a_1\tau + a_0 \,. \qquad [6]$$

By comparing relations [5] and [6] we can clearly see that the acceleration due to gravity corresponds to

$$g = 2a_2 \,, \quad \sigma_g = 2\sigma_{a_2} \,,$$

where σ_g is uncertainty of the g. Coefficient a_1 represents the initial velocity which can be considered close to zero thanks to the construction of the release mechanism.

Coefficient a_0 represents the difference between the real height of the free fall and the value set on the centimeter scale. Since the free fall time depends also on the ball diameter (for the same position of the holder), it is difficult to set real free fall height. Thanks to the least squares method processing we can choose any clearly identifiable place for the height setup (bottom of the holder (8) from the Fig. 3, for example). Correctness of the resulting acceleration due to gravity will not be affected by that, just the coefficient a_0 will be.