Uncertainties of measurement

There are two methods for determination of uncertainty.

Method A – based on mathematical statistics
Method B – based on information given by measuring devices

Method A

This method uses mathematical statistics. An average value is calculated first by the formula.
N is number of measurements
x_i – i-th sample

$$\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i,$$

Then the standard uncertainty equal to the standard deviation of average value is given by

$$u_A = \overline{s} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N(N-1)}}.$$

Method B

For successful calculation of this method we need some data from measuring devices, mostly their precision or resolution. If the resolution of the device is **Δ** (which means one basic division at a needle device, for example), then the B type uncertainty is given by

$$u_B = \frac{\Delta}{\sqrt{12}}.$$

<u>Caliper</u>
Δ = 20 um,

$$u_B = \frac{20\,\mu\text{m}}{\sqrt{12}} \approx 5{,}8\,\mu\text{m}.$$

<u>Micrometer</u>
Δ = 10 um

u_B= 2,9 um

<u>Needle measuring device</u>
This device is characterized by a *class of precision* CP and its *range* R.
Class B uncertainty is given by u_B= (2x(R*CP)/100) / sqrt(12) or u_B= ((R*CP)/100) / sqrt(3)

Example: CP= 0,5 % ; R= 600 mA

$$\pm 600 \times \frac{0{,}5}{100}\,\text{mA} = \pm 3\,\text{mA}.$$

$$u_B = \frac{3}{\sqrt{3}}\,\text{mA} \approx 1{,}7\,\text{mA}.$$

Digital measuring device
Precision is typically given by expression **p**% of rdg + **n** digits

u_B= ((p*rdg)/100 + ls digit value) / sqrt(3)

Example: digital multimeter, full range is 19,99 V; p= 0,5% ; rdg= 12,69V; n= 1 digit; ls digit value= 0,01 V

$$\pm\left(12{,}69\times\frac{0{,}5}{100}+0{,}01\right)\ \mathrm{V}=\pm 0{,}163\ \mathrm{V}.$$

$$u_B=\frac{0{,}163}{\sqrt{3}}\ \mathrm{V}\approx 94\,\mathrm{mV}.$$

Digital device with unknown precision
Precision is commonly considered 2 times the least significant digit

Example: digital counter measuring time, display format is xxxx.xx seconds. The least significant digit is 0,01 s => Δ= 0,02 s

u_B = Δ / sqrt(12) = 0,02 / sqrt (12) = 0,0058 s = 5,8 ms

Time measurement by a stopwatch
A student will estimate his/her precision of measurement.
Example Δ= 0,3 s

u_B = Δ / sqrt(12) = 0,3 / sqrt (12) = 0,087 s = 87 ms

Combined uncertainty for direct measurement – C type uncertainty

Is given by a formula

$$u_C=\sqrt{u_A^2+u_B^2}.$$

Combined uncertainty for a function

$$Z=f(X_1,X_2,\ldots,X_M)$$

$$u_c^2(Y)=\sum_{i=1}^{M}\left(\frac{\partial f}{\partial X_i}\right)^2_{\overline{x}_1,\overline{x}_2,\ldots,\overline{x}_M}u^2(X_i).$$

$$h=\frac{1}{2}gt^2\quad\Rightarrow\quad g=\frac{2h}{t^2},$$

$$\overline{g}=\frac{2\overline{h}}{\overline{t}^2}.$$

$$\frac{\partial g}{\partial h}=\frac{2}{t^2},\qquad\frac{\partial g}{\partial t}=-\frac{4h}{t^3}$$

$$u(g)=\sqrt{\frac{4}{\overline{t}^4}u^2(h)+\frac{16\overline{h}^2}{\overline{t}^6}u^2(t)}.$$