

MEASUREMENT OF VOLUMES

OBJECT

1. To determine the volume of a prism and a cylinder using a caliper and a micrometer.
2. To calculate combined uncertainty of particular dimensions of examined body.
3. To calculate volume of both bodies and combined uncertainties of those volumes.

THEORY

Volumes of cylinder and prism can be evaluated by:

$$V_C = \frac{1}{4}\pi d^2 h = f(d, h) \quad \text{and} \quad V_P = abc = f(a, b, c),$$

where d is diameter of a cylinder, h is height of a cylinder and a, b, c , are dimension of sides of a prism.

For the most probable values of volumes we can write

$$\bar{V}_C = \frac{1}{4}\pi \bar{d}^2 \bar{h} \quad \text{and} \quad \bar{V}_P = \bar{a}\bar{b}\bar{c},$$

where the horizontal line represents an arithmetic average of each measured value.

We know, that the combined uncertainty for a function Y of more variables can be calculated by

$$u_c^2(Y) = \sum_{i=1}^M \left(\frac{\partial f}{\partial X_i} \right)_{\bar{X}_1, \bar{X}_2, \dots, \bar{X}_M}^2 u^2(X_i) \quad \text{where } Y(X_1, X_2, \dots, X_M) \text{ is a function of } M \text{ variables}$$

If we apply this to our volume of a cylinder, we obtain the combined uncertainty

$$u_c(\bar{V}_C) = \sqrt{\frac{\pi^2 \bar{d}^4}{16} u_c^2(\bar{h}) + \frac{\pi^2 \bar{d}^2 \bar{h}^2}{4} u_c^2(\bar{d})},$$

and similarly we can write for the combined uncertainty of a volume of a prism

$$u_c(\bar{V}_P) = \sqrt{(\bar{b}\bar{c})^2 u_c^2(\bar{a}) + (\bar{a}\bar{c})^2 u_c^2(\bar{b}) + (\bar{a}\bar{b})^2 u_c^2(\bar{c})},$$

where $u_c(a)$, $u_c(b)$, $u_c(c)$, $u_c(h)$, $u_c(d)$, are combined uncertainties for particular dimensions.

How to determine combined uncertainties for particular dimensions?

If a measurement of quantity x is repeated n times, then measured values form a set of x_1, x_2, \dots, x_n . The most probable value of the quantity is represented by the average

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

Standard A type uncertainty of the average can be calculated by

$$u_A(\bar{x}) = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Standard B type uncertainty we estimate by the relation

$$u_B = \frac{\Delta}{\sqrt{12}},$$

where the Δ is the least division on the scale of the measuring device. For example, in case of the caliper the $\Delta = 20\mu\text{m} = 0.02\text{mm}$ and in case of the micrometer the $\Delta = 10\mu\text{m} = 0.01\text{mm}$.

Combined uncertainty we obtain from

$$u_C(\bar{x}) = \sqrt{u_A^2(\bar{x}) + u_B^2}$$

The final result of the particular dimension should be recorded in the form

$$x = \bar{x} \pm u_C(\bar{x})$$

and final result of the volume similarly

$$V = \bar{V} \pm u_C(\bar{V})$$

In recording and presenting the results of data it is important to use figures that indicate the size of the measured quantity as well as the accuracy of the information. The number of significant figures is not governed by the position of the decimal point in the recorded result. The location of the decimal point is determined by the size of the units used. For example - the length measured to be (97.3 ± 0.1) cm can be recorded as $(973 \pm 1)\text{mm}$ or (0.973 ± 0.001) m.

In practice the probable error in the result is evaluated in terms of one or two significant figures. For example - the number 6.025 has four significant figures. But the numbers 0.25; 0.025; 0.00025 - each of them has two significant figures only.

If the error is evaluated in terms of one significant figure the mean value of the quantity is completed at the same place; for example

$$\begin{aligned} f &= (23.49 \pm 0.06) \text{ s}^{-1} \\ G &= (327 \pm 4) \text{ N} \\ m &= (38.4 \pm 0.2) \text{ kg} \\ P &= (960 \pm 10) \text{ W} \\ c &= (2.99790 \pm 0.00006) \cdot 10^8 \text{ ms}^{-1} \end{aligned}$$

Lets compare the correct and incorrect records:

incorrect record	correct record
14.5 ± 0.04	14.50 ± 0.04
0.68 ± 0.2	0.7 ± 0.2
0.3322 ± 0.0672	0.33 ± 0.07

If the error is evaluated in terms of two significant figures the mean value is completed at the place that conforms to the place of the second significant figure of the error; for example

$$\begin{aligned} t &= (23.494 \pm 0.064) \text{ s} \\ G &= (327.0 \pm 4.3) \text{ N} \\ m &= (38.35 \pm 0.22) \text{ kg} \end{aligned}$$

Lets compare the correct and incorrect records:

incorrect record	correct record
29.32 ± 0.068	29.325 ± 0.068
0.6836 ± 0.096	0.689 ± 0.096
658.71 ± 3.872	658.7 ± 3.9

PROCEDURE

1. The measurement of each dimension must be done 10 times to be able to obtain a set of values x_1, x_2, \dots, x_{10} .
2. Calculate an average and uncertainties u_A, u_B and u_C for each dimension
3. Calculate the volume of each body and its combined uncertainty